Solving the Static Design Routing and Wavelength Assignment Problem

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Outline

1. Problem
2. Complete Model Variants
3. Decomposition
4. Experimental Results

Main Points

- Compare static design and demand acceptance versions of RWA
- See impact of objective function
- Compare finite domain, MIP and SAT solutions

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Problem Definition

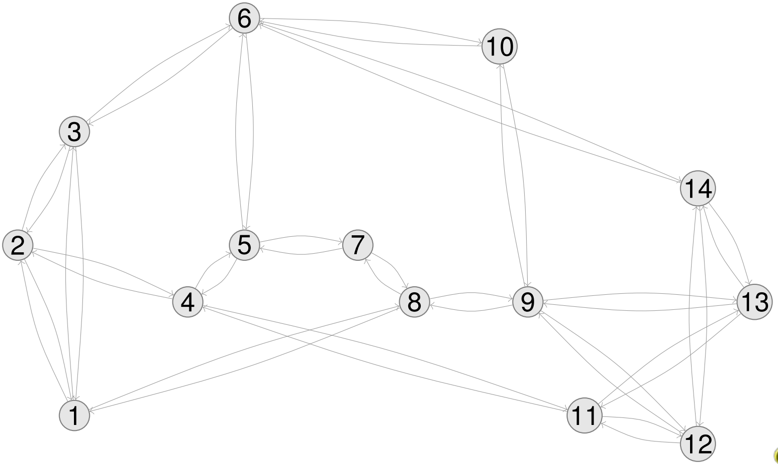
Routing and Wavelength Assignment (Static Design)

In an optical network, traffic demands between nodes are assigned to a route through the network and a specific wavelength. The route (called *lightpath*) must be a simple path from source to destination. Demands which are routed over the same link must be allocated to different wavelengths, but wavelengths may be reused for demands which do not meet. The objective is to find a combined routing and wavelength assignment which minimizes the number of wavelengths used for a given set of demands.

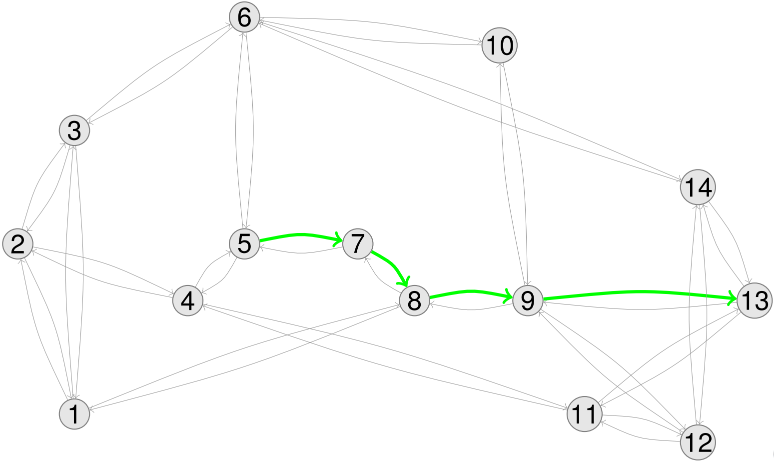
RWA Problem Variants

- Static design
  - Accept all demands
  - Minimize frequencies required
  - Design problem
- Demand acceptance
  - Number of frequencies fixed
  - Maximize number of demands accepted
  - Operational problem

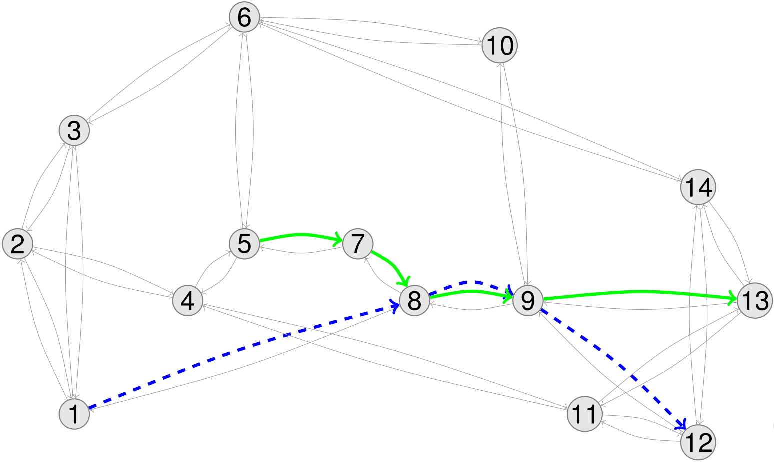
Example Network (NSF, 14 nodes)



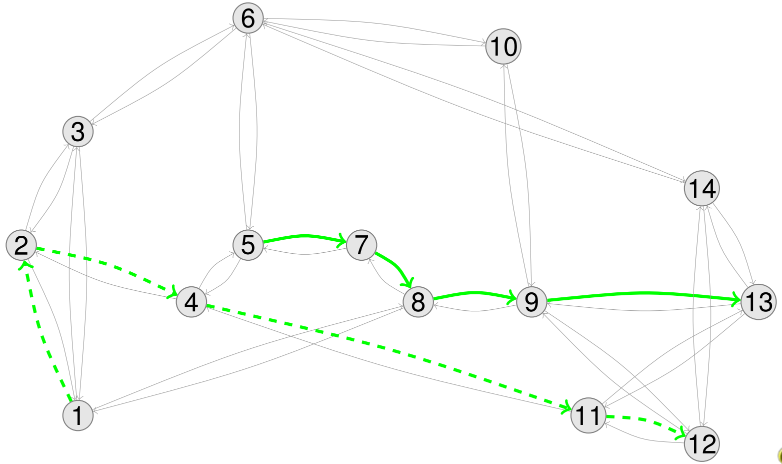
Lightpath from node 5 to node 13 ($5 \Rightarrow 13$)



Conflict with demand 1 ⇒ 12: Use different frequencies



Conflict with demand 1 ⇒ 12: Use different path



Solution Approaches

- Greedy heuristic
- Optimization algorithm for complete problem
- Decomposition into two problems
  - Find routing
  - Assign wavelengths

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- Greedy heuristic
- Optimization algorithm for complete problem
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1. Problem
2. **Complete Model Variants**
3. Decomposition
4. Experimental Results

What is the Objective?

- Basic model
  - Minimize number of frequencies used on any link
  - Cost of equipment (?)
- Extended model
  - Minimize overall number of frequencies
  - Cost of renting fibres (?)

Notation

- Network (N,E) directed graph with nodes N and edges E
- Demands D from source $s(d)$ to sink $t(d)$
- $\mathrm{Out}(n)$ all links leaving n, $\mathrm{In}(n)$ all links entering n
- Available frequencies Λ
- 0/1 integer variables x_{de}^{λ}, demand d is routed over edge e using frequency λ
- 0/1 integer variables y_{d}^{λ}, demand d is using frequency λ

Basic Model

$$\min \max_{e \in E} \sum_{d \in D, \lambda \in \Lambda} x_{de}^{\lambda}$$

s.t.

$$\begin{aligned}
y_d^{\lambda} \in \{0,1\}, x_{de}^{\lambda} &\in \{0,1\} \\
\forall d \in D: \quad \sum_{\lambda \in \Lambda} y_d^{\lambda} &= 1 \\
\forall e \in E, \forall \lambda \in \Lambda: \quad \sum_{d \in D} x_{de}^{\lambda} &\leq 1 \\
\forall d \in D, \forall \lambda \in \Lambda: \quad \sum_{e \in \mathrm{In}(s(d))} x_{de}^{\lambda} = 0, \sum_{e \in \mathrm{Out}(s(d))} x_{de}^{\lambda} &= y_d^{\lambda} \\
\forall d \in D, \forall \lambda \in \Lambda: \quad \sum_{e \in \mathrm{Out}(t(d))} x_{de}^{\lambda} = 0, \sum_{e \in \mathrm{In}(t(d))} x_{de}^{\lambda} &= y_d^{\lambda} \\
\forall d \in D, \forall \lambda \in \Lambda, \forall n \in N \setminus \{s(d), t(d)\}: \quad \sum_{e \in \mathrm{In}(n)} x_{de}^{\lambda} &= \sum_{e \in \mathrm{Out}(n)} x_{de}^{\lambda}
\end{aligned}$$

Extended Model, Additional Variables

- Minimize the total number of variables used
- 0/1 integer variables z^{λ}, frequency λ is used by at least one demand

Extended Model

$$\min \sum_{\lambda \in \Lambda} z^{\lambda}$$

s.t.

$$z^{\lambda} \in \{0,1\}, y_d^{\lambda} \in \{0,1\}, x_{de}^{\lambda} \in \{0,1\}$$

$$\forall d \in D : \quad \sum_{\lambda \in \Lambda} y_d^{\lambda} = 1$$

$$\forall d \in D, \forall e \in E, \forall \lambda \in \Lambda : \quad x_{de}^{\lambda} \leq z^{\lambda}$$

$$\forall e \in E, \forall \lambda \in \Lambda : \quad \sum_{d \in D} x_{de}^{\lambda} \leq 1$$

$$\forall d \in D, \forall \lambda \in \Lambda : \quad \sum_{e \in \mathrm{In}(s(d))} x_{de}^{\lambda} = 0, \sum_{e \in \mathrm{Out}(s(d))} x_{de}^{\lambda} = y_d^{\lambda}$$

$$\forall d \in D, \forall \lambda \in \Lambda : \quad \sum_{e \in \mathrm{Out}(t(d))} x_{de}^{\lambda} = 0, \sum_{e \in \mathrm{In}(t(d))} x_{de}^{\lambda} = y_d^{\lambda}$$

$$\forall d \in D, \forall \lambda \in \Lambda, \forall n \in N \setminus \{s(d), t(d)\} : \quad \sum_{e \in \mathrm{In}(n)} x_{de}^{\lambda} = \sum_{e \in \mathrm{Out}(n)} x_{de}^{\lambda}$$

Problems

- Scalability
    - Network size
    - Number of demands
- Symmetries in model

Improvement: Source Aggregation

- Combine all demands starting in common source
- Removes some, but not all symmetries
- 0/1 integer variables x_{se}^{λ}, a demand starting in s is routed over edge e using frequency λ
- Integer objective $z_{\max}$
- Integer P_{sd}, number of demands between s and d
- Set D_s, all destinations for demands starting in s

Source Aggregation, Basic Model

$$\min z_{\max}$$

s.t.

$$z_{\max} \in \{0, 1 ... |\Lambda|\}, x_{se}^{\lambda} \in \{0, 1\}$$

$$\forall e \in E, \forall \lambda \in \Lambda : \quad \sum_{s \in N} x_{se}^{\lambda} \leq 1$$

$$\forall s \in N, \forall \lambda \in \Lambda : \quad \sum_{e \in \mathrm{In}(s)} x_{se}^{\lambda} = 0$$

$$\forall s \in N, \forall d \in D_s, \forall \lambda \in \Lambda : \quad \sum_{e \in \mathrm{In}(d)} x_{se}^{\lambda} \geq \sum_{e \in \mathrm{Out}(d)} x_{se}^{\lambda}$$

$$\forall s \in N, \forall d \in D_s : \quad \sum_{\lambda \in \Lambda} \sum_{e \in \mathrm{In}(d)} x_{se}^{\lambda} = \sum_{\lambda \in \Lambda} \sum_{e \in \mathrm{Out}(d)} x_{se}^{\lambda} + P_{sd}$$

$$\forall s \in N, \forall n \neq s, n \notin D_s, \forall \lambda \in \Lambda : \quad \sum_{e \in \mathrm{In}(n)} x_{se}^{\lambda} = \sum_{e \in \mathrm{Out}(n)} x_{se}^{\lambda}$$

$$\forall_{e \in E} : \quad \sum_{s \in N} \sum_{\lambda \in \Lambda} x_{se}^{\lambda} \leq z_{\max}$$

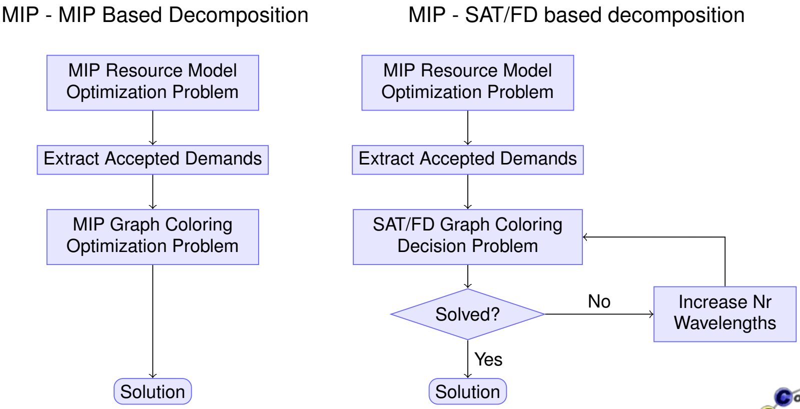
Observations

- Basic model scales reasonably well
- Extended model very poor
    - LP relaxation extremely weak
    - LP bound 1
- Neither works well enough for larger problem sizes
- Aggregated model does not directly provide solution

Outline

1. Problem
2. Complete Model Variants
3. Decomposition
   - Phase 1 MIP
   - Phase 2 MIP
   - Phase 2 Finite Domain Model
   - Phase 2 SAT Model
4. Experimental Results

Solution Approach


MIP - MIP Based Decomposition

MIP Resource Model Optimization Problem

Extract Accepted Demands

MIP Graph Coloring Optimization Problem

Solution

MIP - SAT/FD based decomposition

MIP Resource Model Optimization Problem

Extract Accepted Demands

SAT/FD Graph Coloring Decision Problem

Solved?

No

Increase Nr Wavelengths

Yes

Solution

Idea

- Simplify source aggregation model by ignoring frequencies
- Integer variables z_{se}, how many demands sourced in s are routed over e
- Integer objective $z_{\max}$, corresponds to basic problem
- Constraints independent of number of frequencies, number of demands

Phase 1 MIP

$$\min z_{\max}$$

s.t.

$$z_{\max} \in \{0, 1 \ldots |\Lambda|\}, z_{se} \in \{0, 1 \ldots T_s\}$$

$$\forall s \in N : \quad \sum_{e \in \mathrm{In}(s)} z_{se} = 0$$

$$\forall s \in N, \forall d \in D_s : \quad \sum_{e \in \mathrm{In}(d)} z_{se} = \sum_{e \in \mathrm{Out}(d)} z_{se} + P_{sd}$$

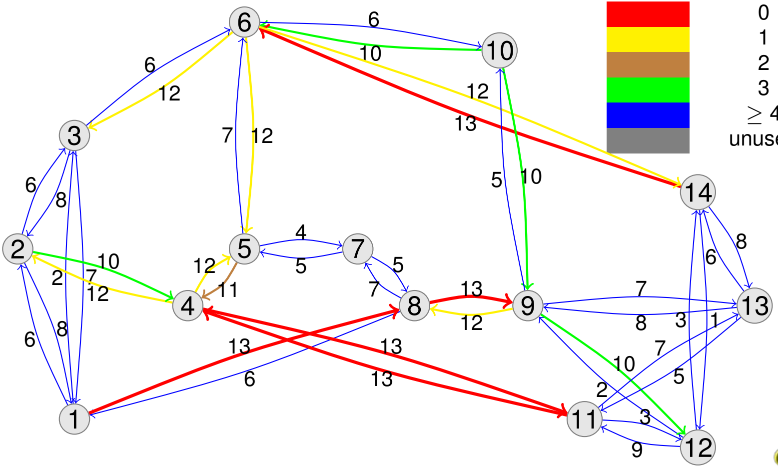
$$\forall s \in N, \forall n \neq s, n \notin D_s : \quad \sum_{e \in \mathrm{In}(n)} z_{se} = \sum_{e \in \mathrm{Out}(n)} z_{se}$$

$$\forall_{e \in E} : \quad \sum_{s \in N} z_{se} \leq z_{\max}$$

Demand Extraction

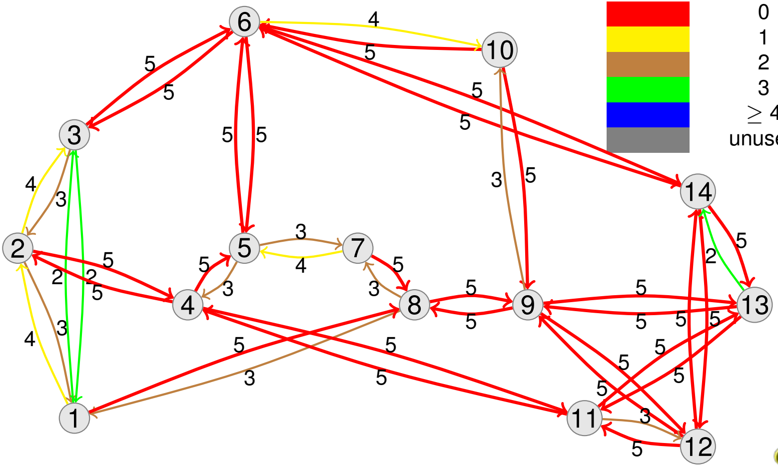
- Find path for each demand
- Non-deterministic, backtrack free search
- Remove loops at same time
- Procedural
- Result: Predicate $p(d, e)$ whether demand d is routed over edge e

Resource Requirements (Static Design)



Color	Off Capacity
🟥	0
🟨	1
🟫	2
🟩	3
🟦	≥ 4
⬜	unused

Compare: Requirements (Demand Acceptance)

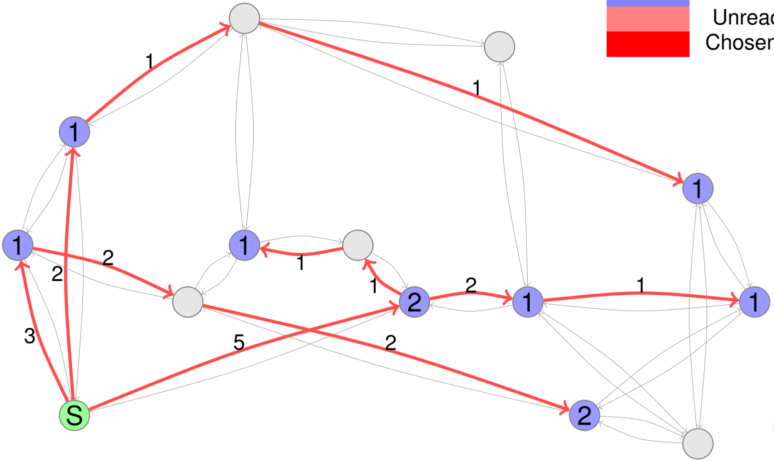


Color	Off Capacity
🟥	0
🟨	1
🟫	2
🟩	3
🟦	≥ 4
▨	unused

Source Model Solution

Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link

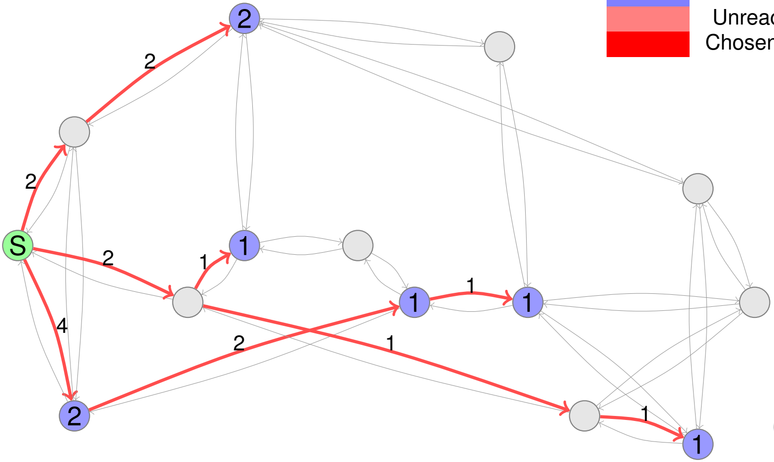
Source Node 1



Source Model Solution

Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link

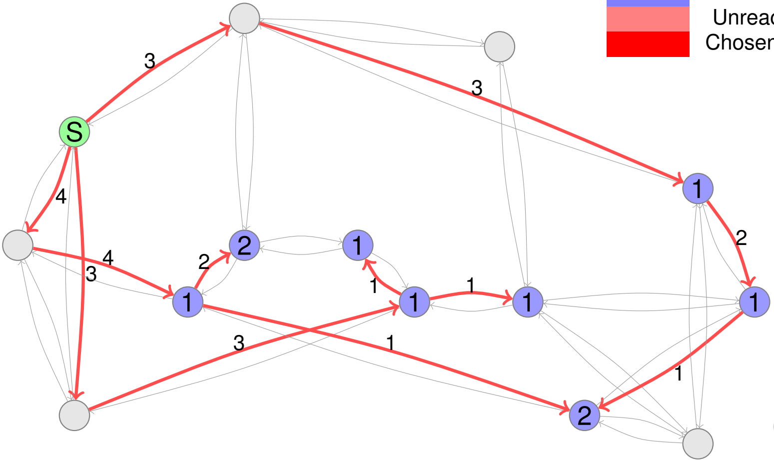
Source Node 2



Source Model Solution

Source Node 3

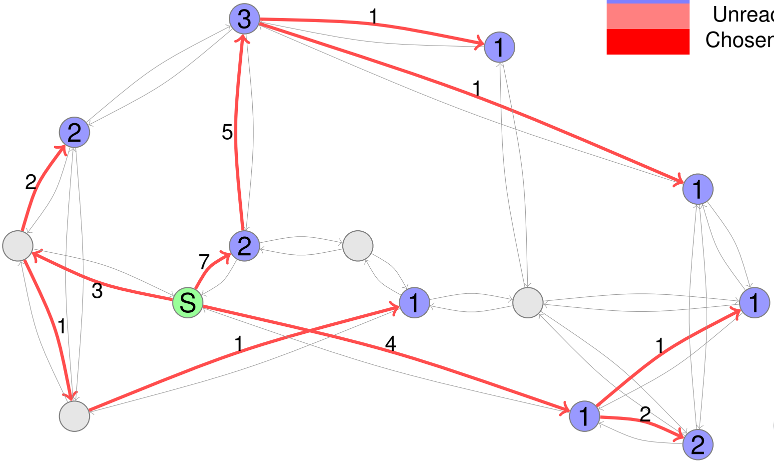
Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link



Source Model Solution

Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link

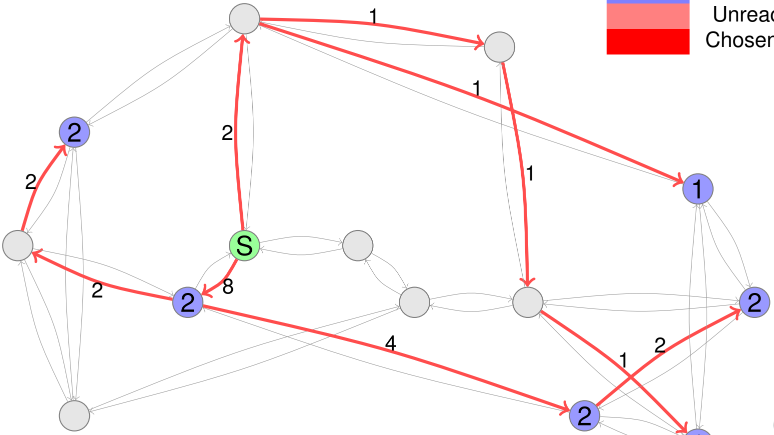
Source Node 4



Source Model Solution

Source Node 5

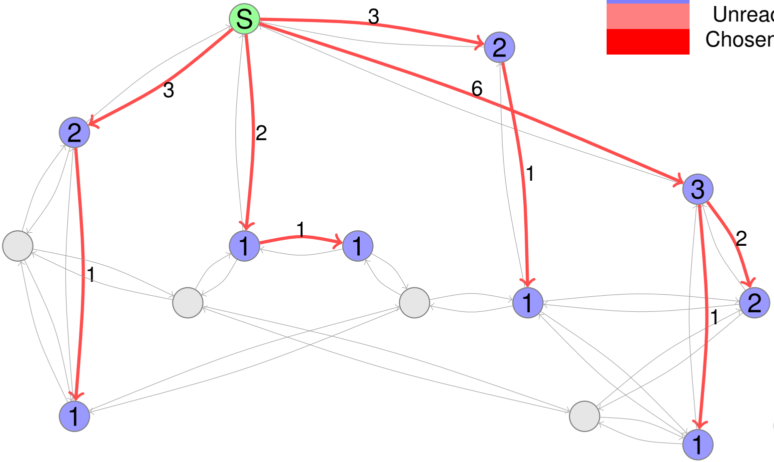
Color	Type
🟩	Source
🟦	Sink
▨	Unreached
🟥	Chosen Link



Source Model Solution

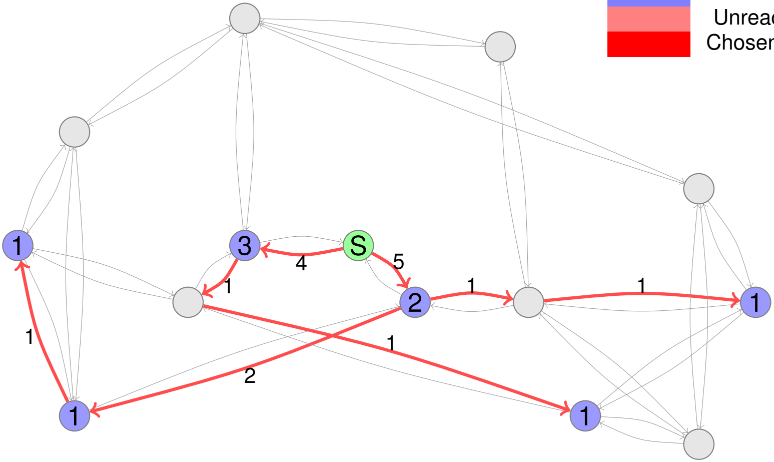
Source Node 6

Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link



Source Model Solution

Source Node 7

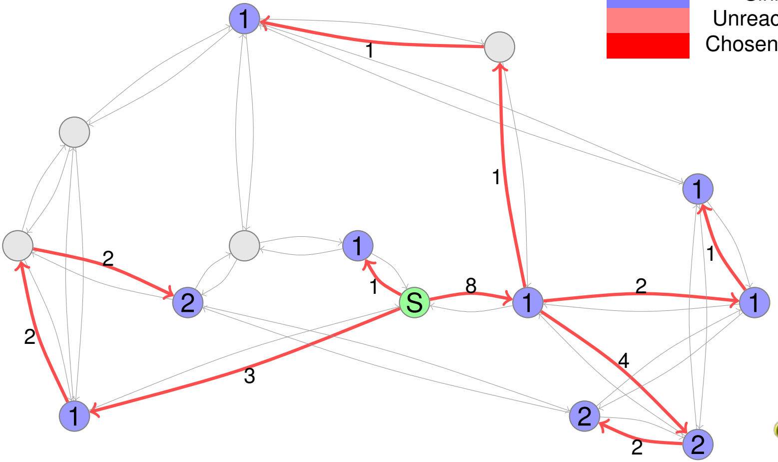


Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link

Source Model Solution

Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link

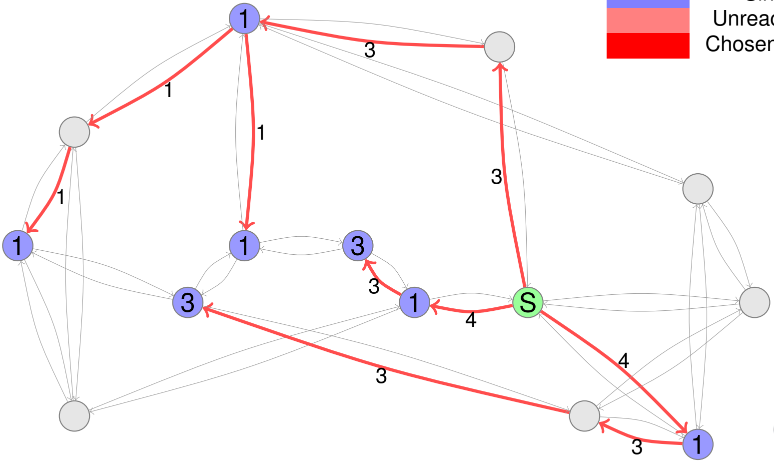
Source Node 8



Source Model Solution

Source Node 9

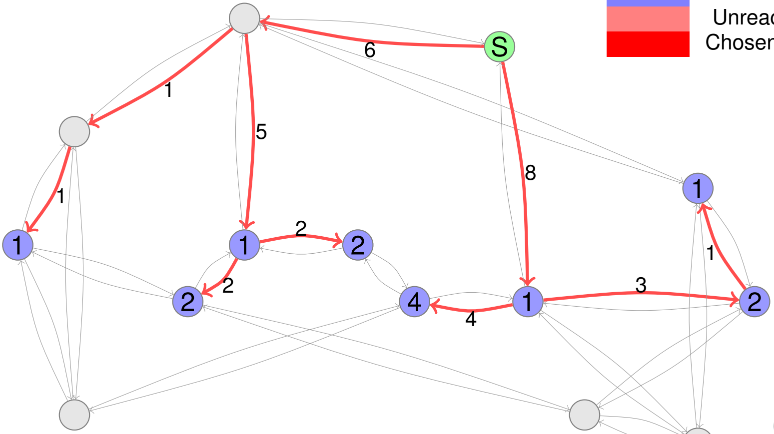
Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link



Source Model Solution

Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link

Source Node 10



Source Model Solution

Color	Type
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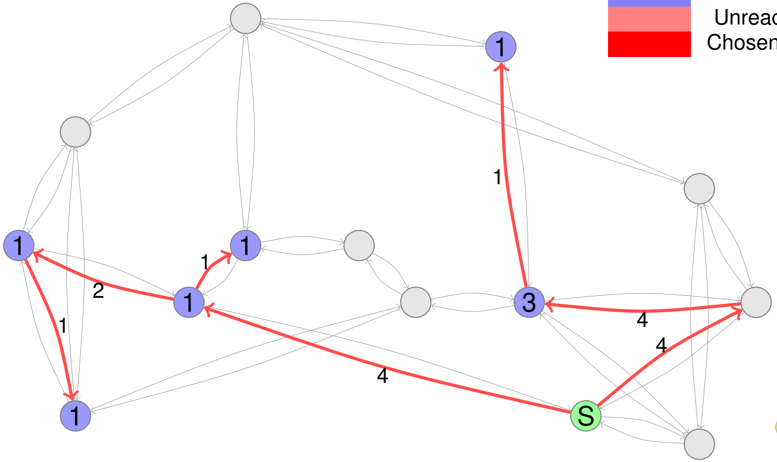
🟦	Sink
▨	Unreached
🟥	Chosen Link

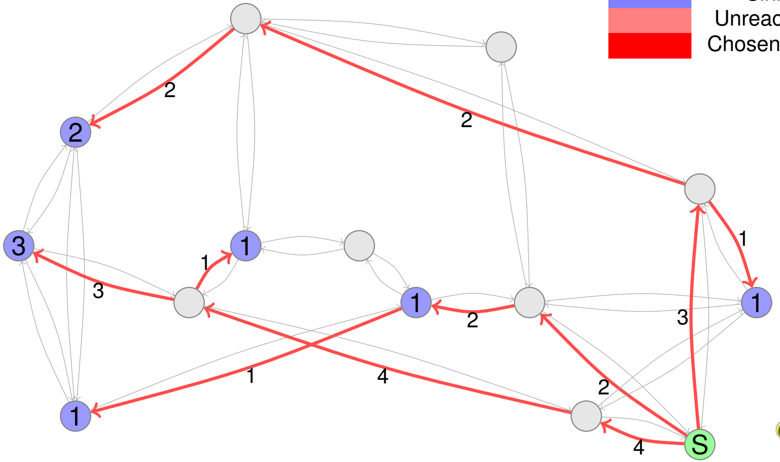
Source Node 11



Source Model Solution

Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link

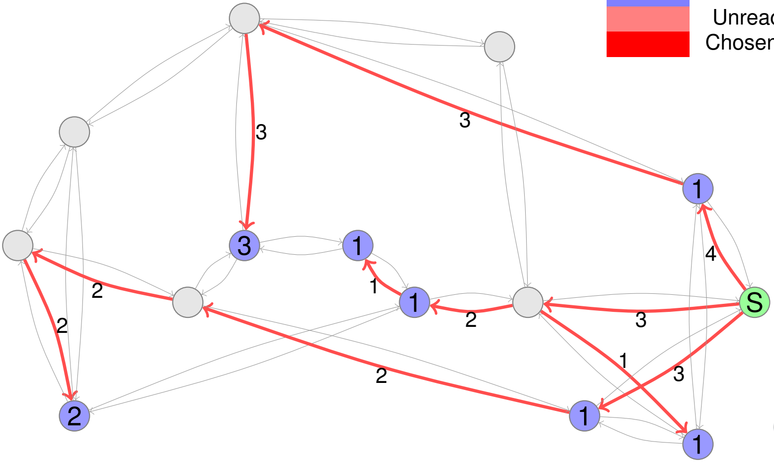
Source Node 12



Source Model Solution

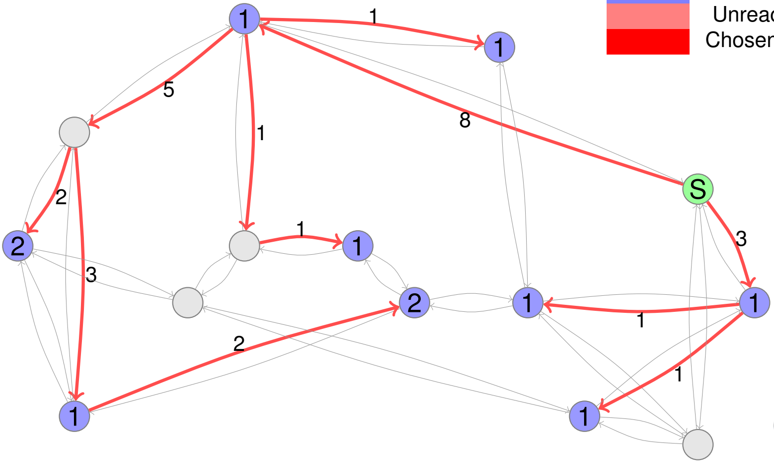
Source Node 13

Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link



Source Model Solution

Source Node 14

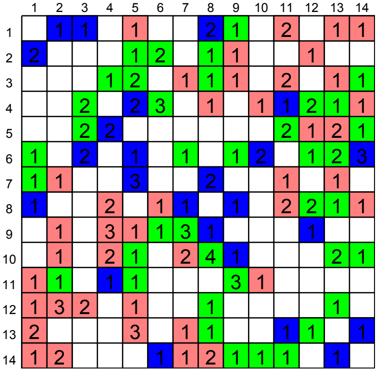


Color	Type
🟩	Source
🟦	Sink
🟥	Unreached
🟥	Chosen Link

Comparison

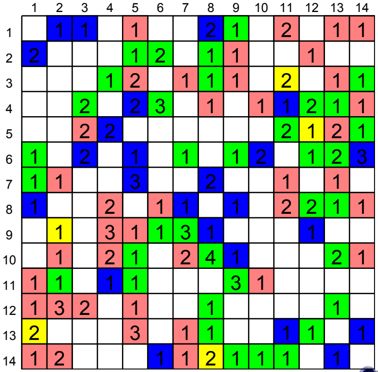
🟦 1 🟩 2 🟥 3 🟨 4

Shortest Path



	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		🟦1	🟦1		🟥1			🟦2	🟩1		🟥2		🟥1	🟥1
2	🟦2				🟩1	🟩2		🟩1	🟥1			🟥1		
3				🟩1	🟩2		🟥1	🟩1	🟥1		🟥2		🟥1	🟩1
4			🟩2		🟦2	🟩3		🟥1		🟥1	🟦1	🟩2	🟩1	🟥1
5			🟩2	🟦2							🟩2	🟥1	🟥2	🟩1
6	🟩1		🟦2		🟦1		🟩1		🟩1	🟦2		🟩1	🟩2	🟦3
7	🟩1	🟥1			🟦3			🟦2			🟥1		🟥1	
8	🟦1			🟥2		🟥1	🟦1		🟦1		🟥2	🟩2	🟩1	🟥1
9		🟥1		🟥3	🟥1	🟩1	🟩3	🟦1				🟦1		
10		🟥1		🟥2	🟩1		🟥2	🟩4	🟦1				🟩2	🟩1
11	🟥1	🟩1		🟦1	🟩1				🟩3	🟥1				
12	🟥1	🟥3	🟥2		🟥1			🟩1					🟩1	
13	🟥2				🟥3		🟥1	🟩1			🟦1	🟩1		🟦1
14	🟥1	🟥2				🟦1	🟥1	🟥2	🟩1	🟩1	🟩1		🟦1	

Routed Demands



	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1		🟦1	🟦1		🟥1			🟦2	🟩1		🟥2		🟥1	🟥1
2	🟦2				🟩1	🟩2		🟩1	🟥1			🟥1		
3				🟩1	🟥2		🟥1	🟩1	🟥1		🟨2		🟥1	🟩1
4			🟩2		🟦2	🟩3		🟥1		🟥1	🟦1	🟩2	🟩1	🟥1
5			🟥2	🟦2							🟩2	🟨1	🟥2	🟩1
6	🟩1		🟦2		🟦1		🟩1		🟩1	🟦2		🟩1	🟩2	🟦3
7	🟩1	🟥1			🟦3			🟦2			🟥1		🟥1	
8	🟦1			🟥2		🟥1	🟦1		🟦1		🟥2	🟩2	🟩1	🟥1
9		🟨1		🟥3	🟥1	🟩1	🟩3	🟦1				🟦1		
10		🟥1		🟥2	🟩1		🟥2	🟩4	🟦1				🟩2	🟥1
11	🟥1	🟩1		🟦1	🟩1				🟩3	🟥1				
12	🟥1	🟥3	🟥2		🟥1			🟩1					🟩1	
13	🟨2				🟥3		🟥1	🟩1			🟦1	🟩1		🟦1
14	🟥1	🟥2				🟦1	🟥1	🟨2	🟩1	🟩1	🟩1		🟦1	

Phase 2 MIP Idea

- Assign each demand to a frequency
- Basic model: Minimize the largest number of frequencies used on any link
- Extended model: Minimize total number of frequencies
- 0/1 integer variables x_d^λ, whether demand d uses frequency λ
- Extended model only: 0/1 integer variables z^λ
- Clash constraints: Only one demand can use each frequency on a link

Phase 2 MIP Model (Basic Problem)

$$\min z_{\max}$$

s.t.

$$x_d^\lambda \in \{0,1\}, z_{\max} \in \{0,1,...,|\Lambda|\}$$

$$\forall_{d \in D}: \quad \sum_{\lambda \in \Lambda} x_d^\lambda = 1$$

$$\forall_{e \in E} \forall_{\lambda \in \Lambda}: \quad \sum_{\{d \in D \mid p(d,e)\}} x_d^\lambda \leq 1$$

$$\forall_{e \in E}: \quad \sum_{\lambda \in \Lambda} \sum_{\{d \in D \mid p(d,e)\}} x_d^\lambda \leq z_{\max}$$

Phase 2 MIP Model (Extended Problem)

$$\min z_{\max}$$

s.t.

$$x_d^\lambda \in \{0,1\}, z^\lambda \in \{0,1\}, z_{\max} \in \{0,1,...,|\Lambda|\}$$

$$\forall_{d \in D}: \quad \sum_{\lambda \in \Lambda} x_d^\lambda = 1$$

$$\forall_{e \in E} \forall_{\lambda \in \Lambda}: \quad \sum_{\{d \in D \mid p(d,e)\}} x_d^\lambda \leq 1$$

$$\forall_{e \in E}: \quad \sum_{\lambda \in \Lambda} \sum_{\{d \in D \mid p(d,e)\}} x_d^\lambda \leq z_{\max}$$

$$\forall_{d \in D} \forall_{\lambda \in \Lambda}: \quad x_d^\lambda \leq z^\lambda$$

$$\sum z^\lambda \leq z_{\max}$$

Finite Domain Model, Idea

- Graph coloring problem
- Finite domain variables y_d, demand d is assigned to frequency y_d
- Two demands routed over same edge must use different frequencies
  - Binary disequality constraints
  - Aggregated to `alldifferent` constraints
- Finite domain variables n_e, edge e uses n_e frequencies
- `nvalue` constraint counts number of different values used

Phase 2 Finite Domain Constraints (Basic Model)

$$\min \max_{e \in E} n_e$$

s.t.

$$y_d \in \{0, 1 ..., |\Lambda|\}, n_e \in \{0, 1 ..., |\Lambda|\}$$

$$\forall_{e \in E} : \quad \mathrm{nvalue}(n_e, \{y_d \mid p(d, e)\})$$

$$\forall_{e \in E} : \quad \mathrm{alldifferent}(\{y_d \mid p(d, e)\})$$

Simplification

- `nvalue` and `alldifferent` constraints are over same variable sets
- Values of `alldifferent` constraint must be different
- `nvalue` constraints can be removed
- n_e variables are not required
- Not an optimization problem!

Simplified Basic Model

$$y_d \in \{0, 1 ..., |\Lambda|\}$$

$$\forall_{e \in E} : \quad \text{alldifferent}(\{y_d \mid p(d, e)\})$$

Phase 2 Finite Domain Constraints (Extended Model)

$$\min \max_{d \in D} y_d$$

s.t.

$$y_d \in \{0, 1 ..., |\Lambda|\}$$

$$\forall_{e \in E} : \quad \mathrm{alldifferent}(\{y_d \mid p(d, e)\})$$

Optimization from below

- Start with known lower bound
- Test value to see if problem is feasible
- If successful, optimal solution reached
- Otherwise increase bound by one and repeat

Phase 2 Finite Domain Simplified Extended Model

$$y_d \in \{0, 1..., C\}$$

$$\forall_{e \in E} : \quad \mathrm{alldifferent}(\{y_d \mid p(d, e)\})$$

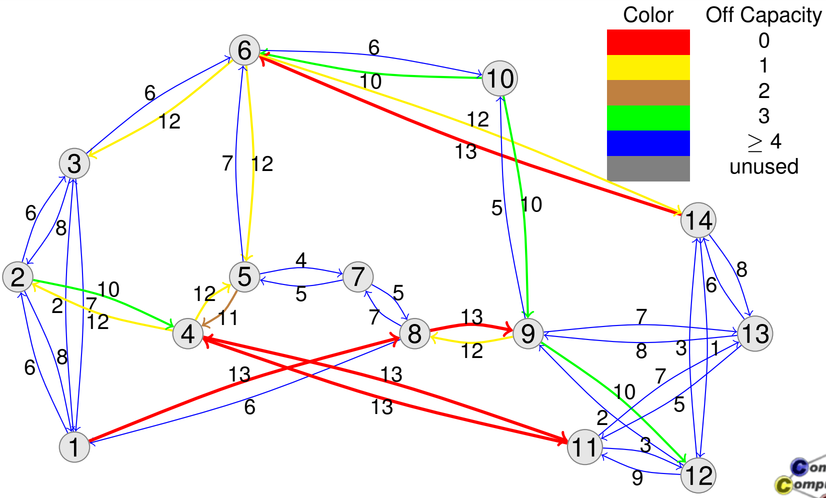
SAT Model

$$\begin{aligned}
\forall_{d \in D} \forall_{\lambda_1, \lambda_2 \in \Lambda \text{ s.t. } \lambda_1 \neq \lambda_2} &: \quad \neg x_d^{\lambda_1} \vee \neg x_d^{\lambda_2} \\
\forall_{d \in D} &: \quad \bigvee_{\lambda \in \Lambda} x_d^{\lambda} \\
\forall_{e \in E} \forall_{\lambda \in \Lambda}, d_1, d_2 \in D \text{ s.t. } p(d_1, e) \wedge p(d_2, e) \wedge d_1 \neq d_2 &: \quad \neg x_{d_1}^{\lambda} \vee \neg x_{d_2}^{\lambda}
\end{aligned}$$

SAT Solver

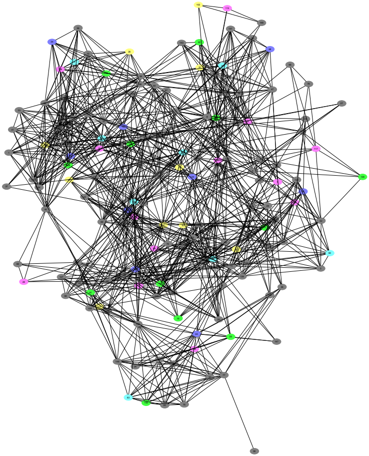
- Use minisat as black box
- Generate problem file in clausal format
- Impose external time limit (100 sec per run)

Recall: Basic Clique Sizes



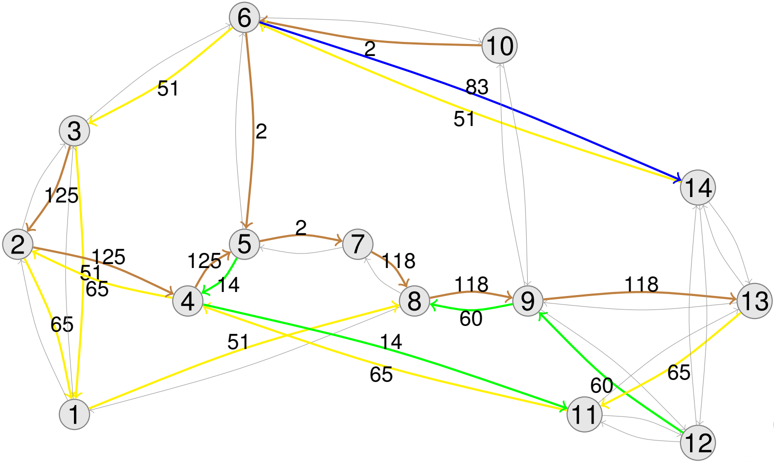
Color	Off Capacity
🟥	0
🟨	1
🟫	2
🟩	3
🟦	≥ 4
▨	unused

Graph Coloring Solution



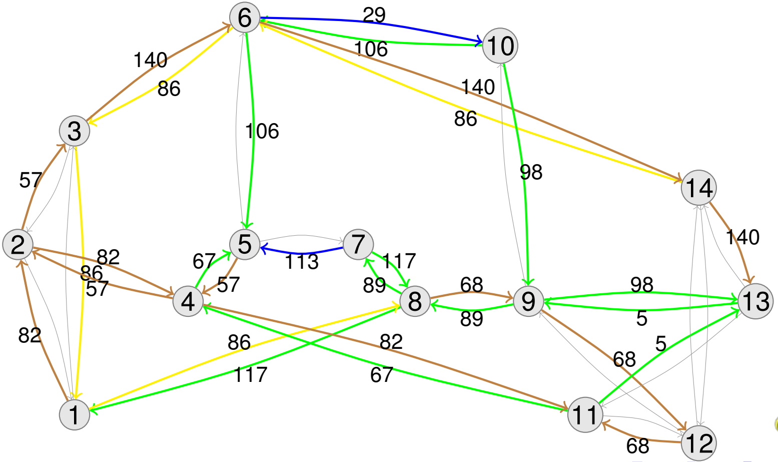
Frequency Assignment

Frequency 1



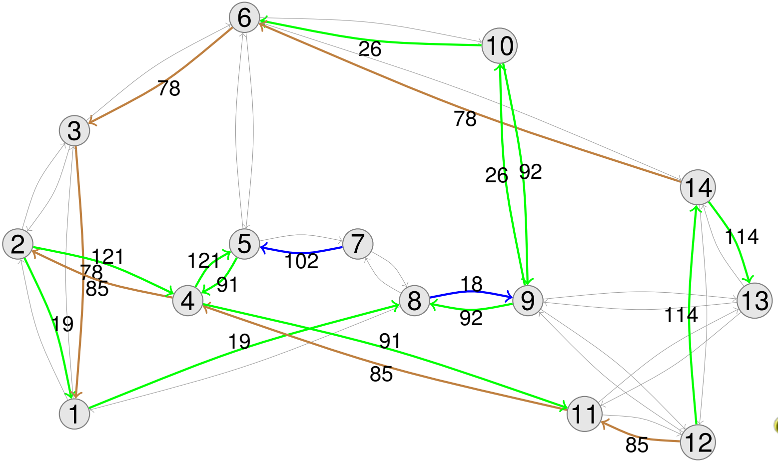
Frequency Assignment

Frequency 2



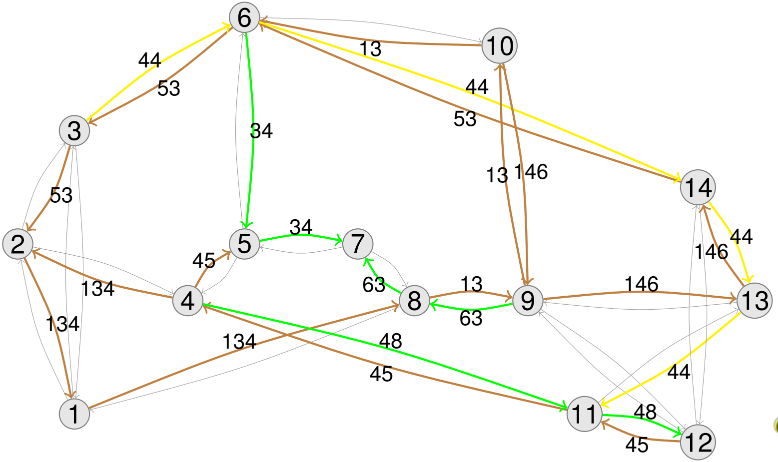
Frequency Assignment

Frequency 3



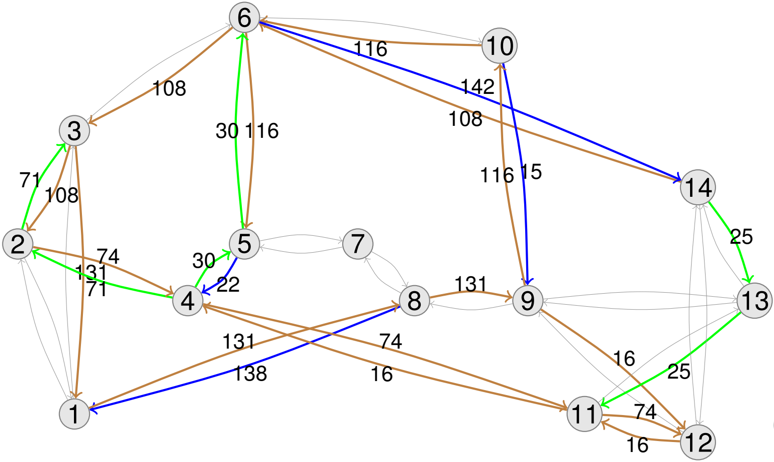
Frequency Assignment

Frequency 4



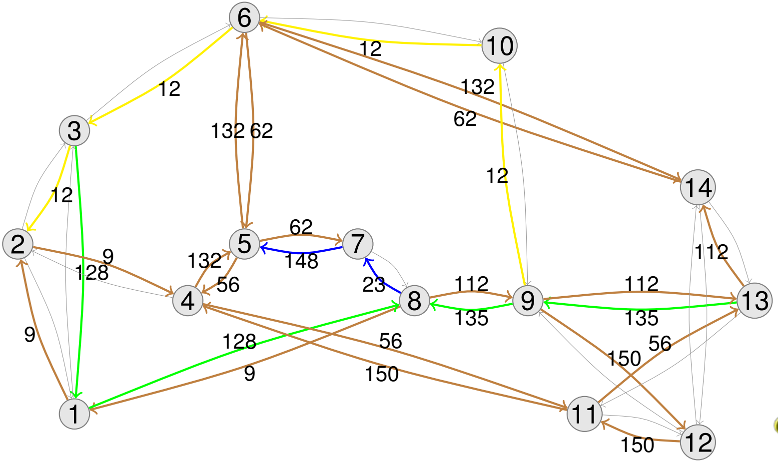
Frequency Assignment

Frequency 5



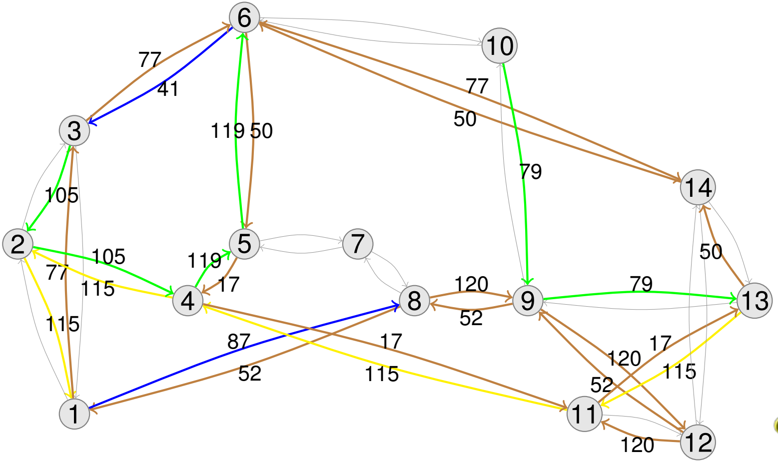
Frequency Assignment

Frequency 6



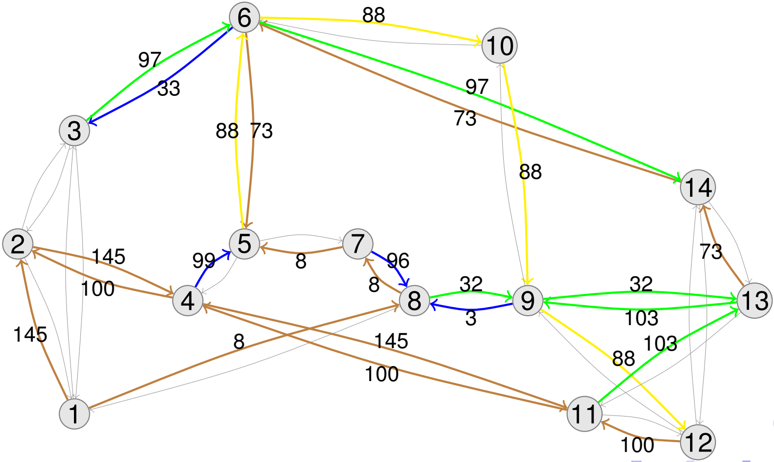
Frequency Assignment

Frequency 7



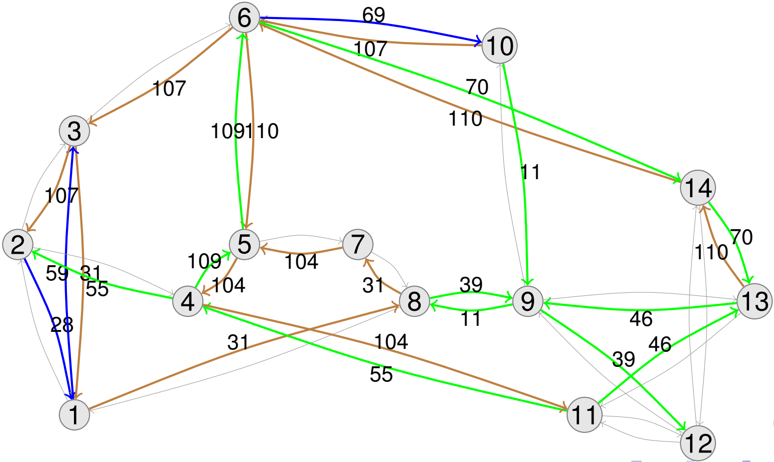
Frequency Assignment

Frequency 8



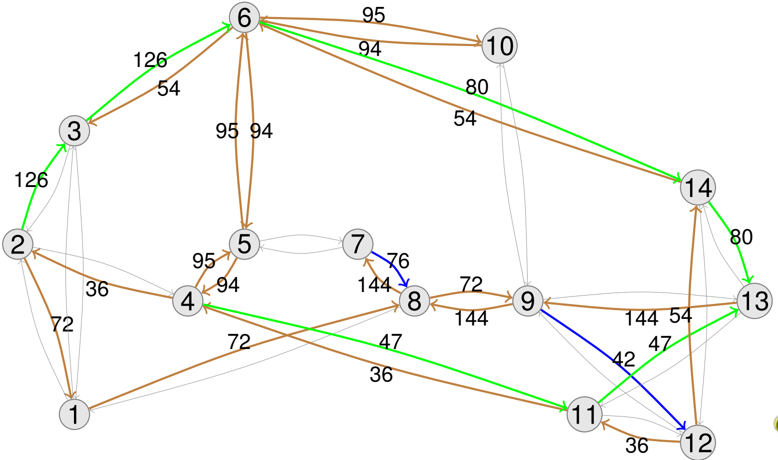
Frequency Assignment

Frequency 9



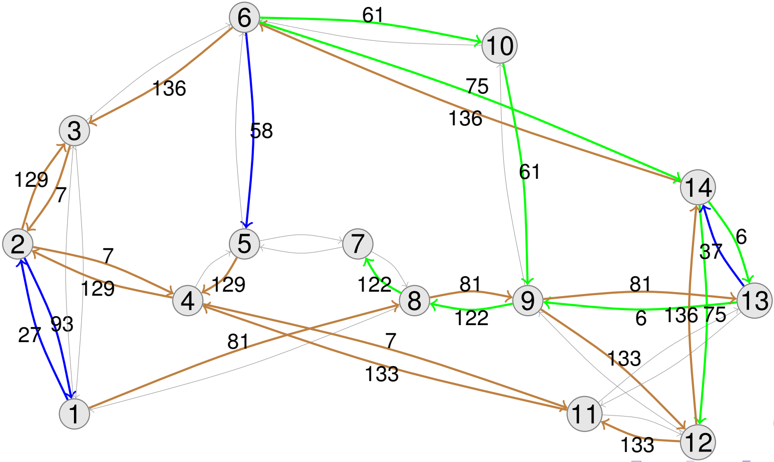
Frequency Assignment

Frequency 10



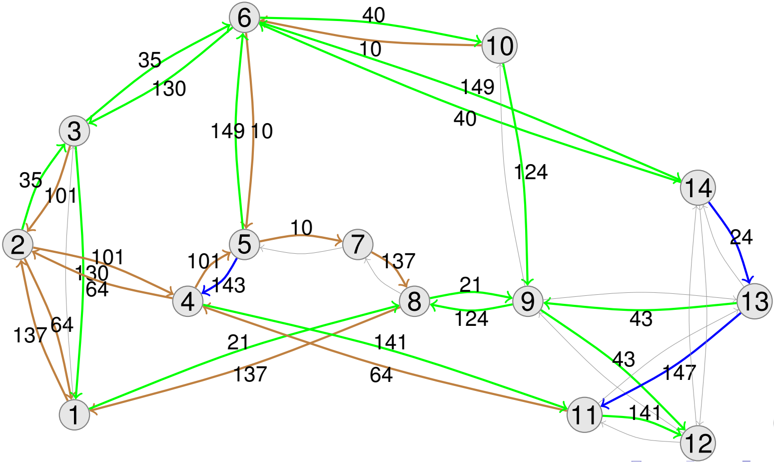
Frequency Assignment

Frequency 11



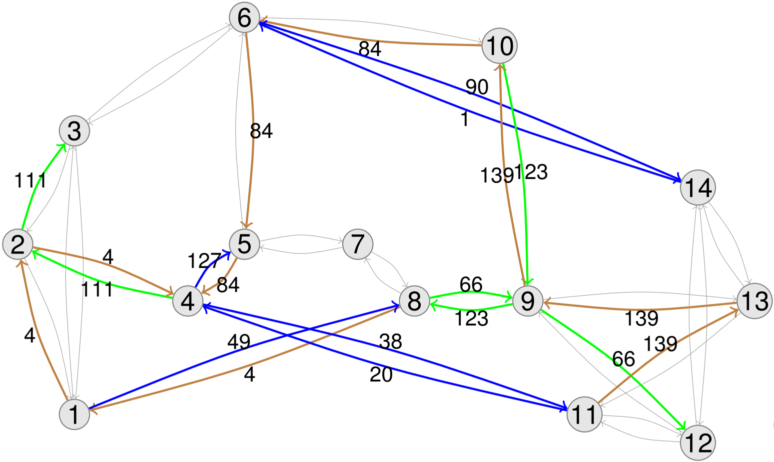
Frequency Assignment

Frequency 12



Frequency Assignment

Frequency 13



Outline

Example Networks

nsf	14 nodes, 42 edges
eon	20 nodes, 78 edges
mci	19 nodes, 64 edges
brezil	27 nodes, 140 edges

Comparison (Basic Problem, 100 Runs Each)

Network	Dem.	Complete MIP		Decomposition					
				MIP-MIP		MIP-FD		MIP-SAT	
		Opt	Avg	Opt	Avg	Opt	Avg	Opt	Avg
brezil	100	100	277.14	100	0.91	100	0.01	100	0.03
brezil	200	-	-	100	4.45	100	0.03	100	0.07
brezil	300	-	-	100	8.08	99	0.07	100	0.15
brezil	400	-	-	100	10.93	100	0.13	100	0.27
brezil	500	-	-	100	13.09	100	0.23	100	0.44
brezil	600	-	-	100	16.77	100	0.31	100	0.69
eon	100	100	33.62	100	1.51	100	0.01	100	0.04
eon	200	100	65.51	100	5.27	100	0.04	100	0.10
eon	300	100	121.27	100	5.60	100	0.09	100	0.24
eon	400	100	116.64	100	7.38	100	0.16	100	0.45
eon	500	100	162.55	100	9.58	100	0.29	100	0.76
eon	600	100	232.91	99	14.04	100	0.40	100	1.20
mci	100	100	20.27	100	2.08	100	0.01	100	0.05
mci	200	100	38.79	100	5.36	100	0.05	100	0.12
mci	300	100	55.78	100	5.83	100	0.10	100	0.29
mci	400	100	109.85	100	8.71	100	0.19	100	0.56
mci	500	100	129.90	100	13.89	100	0.29	100	0.97
mci	600	100	257.70	100	22.56	100	0.45	100	1.55
nsf	100	100	8.17	100	2.38	100	0.02	100	0.05
nsf	200	100	12.75	100	1.81	100	0.05	100	0.15
nsf	300	100	17.01	100	1.98	100	0.10	100	0.35
nsf	400	100	27.36	100	3.54	100	0.17	100	0.71
nsf	500	100	54.60	100	5.77	100	0.31	100	1.26
nsf	600	100	88.72	100	9.09	100	0.43	100	2.07

Comparison (Extended Problem, 100 Runs Each)

		Complete		Decomposition					
Network	Dem.	MIP		MIP-MIP		MIP-FD		MIP-SAT	
		Opt	Avg	Opt	Avg	Opt	Avg	Opt	Avg
brezil	100	-	-	94	53.59	95	0.02	96	0.02
brezil	200	-	-	99	141.04	99	0.06	99	0.06
brezil	300	-	-	88	444.64	99	0.12	98	3.09
brezil	400	-	-	-	-	99	0.23	99	1.21
brezil	500	-	-	-	-	96	0.93	95	7.83
brezil	600	-	-	-	-	97	0.45	82	21.69
eon	100	-	-	100	19.70	100	0.02	100	0.02
eon	200	-	-	100	188.55	100	0.07	100	0.06
eon	300	-	-	-	-	100	0.16	100	0.19
eon	400	-	-	-	-	100	0.26	100	0.57
eon	500	-	-	-	-	100	0.44	87	15.32
eon	600	-	-	-	-	100	0.60	42	66.10
mci	100	-	-	100	26.27	100	0.02	100	0.02
mci	200	-	-	96	271.65	100	0.08	100	0.08
mci	300	-	-	-	-	100	0.17	100	0.27
mci	400	-	-	-	-	100	0.32	97	4.15
mci	500	-	-	-	-	100	0.48	78	24.33
mci	600	-	-	-	-	100	0.68	33	76.84
nsf	100	-	-	99	29.43	99	0.03	99	0.09
nsf	200	-	-	99	208.72	100	0.07	100	0.10
nsf	300	-	-	-	-	100	0.15	100	0.48
nsf	400	-	-	-	-	100	0.26	90	11.46
nsf	500	-	-	-	-	100	0.42	41	70.70
nsf	600	-	-	-	-	100	0.58	23	104.04

Increasing Demand Number (Extended Problem, 100 Runs Each)

Network	Dem.	λ	Opt.	Avg LP	Avg MIP	Avg FD	Max LP Gap	Max FD Gap	Avg MIP Time	Max MIP Time	Avg FD Time	Max FD Time
brezil	700	150	97	25.69	26.06	26.13	0.75	3.00	0.51	0.64	1.83	60.59
brezil	800	150	96	29.34	29.66	29.72	0.75	3.00	0.50	0.59	1.42	60.95
brezil	900	150	98	32.81	33.14	33.17	0.75	2.00	0.50	0.61	1.30	31.36
brezil	1000	150	99	36.34	36.68	36.69	0.75	1.00	0.50	0.63	1.24	2.13
brezil	1100	150	99	39.80	40.16	40.17	0.75	1.00	0.50	0.63	1.49	2.20
brezil	1200	150	99	43.28	43.61	43.62	0.75	1.00	0.50	0.63	2.24	46.16
brezil	1300	150	98	46.54	46.89	46.94	0.75	3.00	0.50	0.61	3.03	64.45
brezil	1400	150	99	49.85	50.21	50.23	0.75	2.00	0.50	0.63	2.79	33.95
brezil	1500	150	99	53.46	53.87	53.89	0.75	2.00	0.50	0.61	3.18	34.47
brezil	1600	150	98	56.95	57.28	57.30	0.75	1.00	0.50	0.59	4.49	72.05
brezil	1700	150	99	60.33	60.65	60.66	0.75	1.00	0.51	0.64	3.61	8.92
brezil	1800	150	99	63.93	64.25	64.26	0.75	1.00	0.51	0.61	4.08	9.49
brezil	1900	150	100	67.41	67.77	67.77	0.75	0.00	0.50	0.61	4.73	10.48
brezil	2000	150	99	70.83	71.09	71.10	0.75	1.00	0.51	0.66	6.05	94.73

Increasing Network Size (Extended Problem, 100 Runs Each)

Network	Dem.	λ	Opt.	Avg LP	Avg MIP	Avg FD	Max LP Gap	Max FD Gap	Avg MIP Time	Max MIP Time	Avg FD Time	Max FD Time
r30	500	30	100	7.81	8.12	8.12	0.97	0.00	1.73	5.92	0.16	0.27
r40	500	30	100	4.14	4.52	4.52	0.92	0.00	12.42	177.45	0.13	0.19
r50	500	30	97	2.39	2.88	2.91	0.95	1.00	77.35	696.73	0.11	0.14
r60	500	30	100	1.57	2.05	2.05	0.86	0.00	127.75	245.25	0.10	0.13

Outline

Conclusions

- Modelling static design variant of RWA
- Choice of objective function important
- Simple MIP-FD decomposition works well
- Very good lower bound from phase 1 MIP/LP
- FD graph coloring model outperforms MIP and SAT variants
- Possible to use specialized graph coloring codes (not tested)
- Conceptually simpler than RWA demand acceptance