Chapter 11: Limits of Propagation (Costas Array)

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ECLiPSe ELearning Overview

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What we want to introduce

- Improving propagation does not always pay
- For some problems, simple backtracking is best
- CP may not always be the best method
- CP should always be fastest way to model problem
- Consider time to target
  - Time required to run program
  - Time required to write program
- Problem: Costas Array (Antenna design, sonar systems)
Costas Array (Wikipedia)

A Costas array (named after John P. Costas) can be regarded geometrically as a set of N points lying on the squares of a NxN checkerboard, such that each row or column contains only one point, and that all of the N(N - 1)/2 vectors between each pair of dots are distinct.
- A variable for each column, ranging from 1 to $N$
- A list of $N$ variables for the columns
- A difference variable between each ordered pair of variables
- `alldifferent` constraint between variables
- `alldifferent` constraints for all differences
Example

Declarations

:-module(costas).
:-export(top/0).
:-lib(ic).
Main Program

top:-
   (for(N,3,20) do
      costas(N,_)
   )

costas(N,L):-
   length(L,N),
   L :: 1..N,
   alldifferent(L),
   L = [_|L1],
   diffs(L,L1),
   search(L,0,first_fail,indomain,
       complete,[]).

Differences

diffs(_,[]).

diffs(L,[H|T]):-
   diff_pairs(L,[H|T],Diffs),
   alldifferent(Diffs),
   diffs(L,T).

diff_pairs(_,[],[]).

diff_pairs([X|X1],[Y|Y1],[D|D1]):-
   X #= Y+D,
   diff_pairs(X1,Y1,D1).
Basic Model

Other Problem Sizes

<table>
<thead>
<tr>
<th>Size</th>
<th>Backtrack</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>0.00</td>
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<tr>
<td>11</td>
<td>118</td>
<td>0.08</td>
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<td>12</td>
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</tr>
<tr>
<td>18</td>
<td>115745</td>
<td>283.97</td>
</tr>
</tbody>
</table>
Search tree (Size 14)

Search tree (Size 15)
Search tree (Size 16)

Observation

- Problem becomes harder with increasing size
- Failures occur from level 3 down
- Deep backtracking required to undo wrong choices
- Value selection not working, have to explore all choices
- Increase not uniform
The model is doing this

It could be doing that!
```prolog
diffs(_, []).  
diffs(L, [H|T]) :-  
    diff_pairs(L, [H|T], Diffs, Triples),  
    impose_triples(Triples, []),  
    alldifferent(Diffs),  
    diffs(L, T).

diff_pairs(_, [], [], []).  
diff_pairs([X|X1], [Y|Y1], [D|D1], [t(X,Y,D)|T]) :-  
    X #= Y+D,  
    diff_pairs(X1, Y1, D1, T).

impose_triples([], _).  
impose_triples([t(X,Y,D)|R], Others) :-  
    suspend(impose_triple(D,R), 4, D->inst),  
    suspend(impose_triple(D,Others), 4, D->inst),  
    impose_triples(R, [t(X,Y,D)|Others]).
```
Changed Differences

impose_triple(_D,[]).
impose_triple(D,[t(X,Y,_)|R]):-
    (var(X) ->
        suspend(impose_one_triple12(D,X,Y),
        4,X->inst)
    ;
    impose_one_triple12(D,X,Y)
),
%
    ... 
impose_triple(D,R).
impose_one_triple12(D,X,Y):-
    V is X-D,
    Y #\= V.

Further Model Improvements

- DC consistent alldifferent between variables
- (DC consistent alldifferent between differences)
- DC difference constraint
Improved Model

Comparison (Solutions)

Initial Model

Improved Model
Comparison (Search Trees)

Initial Model

Improved Model

Search tree (Size 12)
Search tree (Size 15)

Search tree (Size 16)
Comparison (Search Tree, size 16)

Initial Model

Improved Model

Other Problem Sizes

<table>
<thead>
<tr>
<th>Size</th>
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<th>Improved Model</th>
</tr>
</thead>
<tbody>
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Observation

- Changes reduce backtracks by 50%
- But, run times explode
- Being clever does not always pay
- Or, perhaps, we did not make the right improvements?

Change of Search Strategy

- Idea: Make more difficult choices first
- Reorder variables to start from middle
- Assign values starting in middle
### Labeling From Middle

![Diagram of a graph with numbers and relationships between them.]

### Other Problem Sizes

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<th>Improved Model, Middle</th>
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Observation

- Big improvement in backtracks and time
- Not for all problem sizes
- Question: Do we need improvement of model for this to work?
- Experiment: Run changes search routine on basic model

Labeling Basic Model from Middle

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<tr>
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### Comparison: Model Impact

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### Comparison (Search Tree, size 18)

**Initial Model**

**Improved Model**
Observation

- Search strategy does not depend on model
- Variable selection is the same!
- Basic model is about two times faster
- About 50% more backtrack steps
- Again, sometimes reasoning does not pay!
- Better search strategy pays off dramatically

A Different Model

- Model shown is not the only way to express problem
0/1 Models

- SAT (Minisat)
- Pseudo Boolean (Minisat+)
- MIP (Coin-OR)

0/1 Models: Variables

- $X_{i\nu}$: Variable $i$ takes value $\nu$
- $D_{ij\nu}$: Difference between variables $i$ and $j$ is $\nu$
MIP Model: Constraints

- alldifferent between variables
  \[ \sum_i X_{iv} = 1 \]
  \[ \sum_v X_{iv} = 1 \]
- alldifferent between differences
  \[ \sum_v D_{ijv} = 1 \]
  \[ \sum_{i-j=c} D_{ijv} \leq 1 \]
- link between variables and differences
  \[ D_{ijv} = \sum_{v1=v2+v} X_{iv1} X_{jv2} \]

More Information

- [http://www.costasarrays.org/](http://www.costasarrays.org/)