A Tale of Two Puzzles

Experiments in Constraint Modelling

Helmut Simonis

CrossCore Optimization
London
UK

Visit to Monash University

Outline

Outline

Motivation

Don't ask what you can do for constraint programming, ask what constraint programming can do for you!

apologies to J.F. Kennedy

Key Points

- CP faces stiff competition from SAT and MIP
- We are still missing simple things
  - Packaged Search
  - Constraint Interaction
- Too much focus on inventing new technology, not enough on using existing techniques

Based on Joint Work

- Mats Carlsson, SICS, Uppsala, Sweden
- Christian Schulte, KTH, Stockholm, Sweden

Why Puzzles?

- Essence of constraint programming
  - Solvable by propagation alone
  - Designed to show nice interaction of constraints
- Ideal test beds for modelling
  - Can you model the problem at all?
  - Does your system do as well as humans?
  - What can we learn from the human expert?
- They are *fun*!

Related Work

- Sudoku as a Constraint Problem (Simonis, 2005)
- Solitaire Battleship (Smith, 2006)
- Teaching constraints through logic puzzles (Szeredi, 2003)
- Crossword Puzzles (Van Hentenryck 89, LeProvost&Wallace 92)

Some Definitions

Definition

A puzzle is called *valid* if it has solutions.

Definition

A puzzle is called *well posed* if it has exactly one solution.

Disclaimer

- Work in progress
- Use of MILP and SAT strictly naive
  - Eplex solver in ECLiPSe (Shen & Schimpf, 2005)
  - (Cplex 10.1 results provided by K. Shen)
  - Minisat+ and Minisat 2.0 (Een & Srensson, 2003)

Outline

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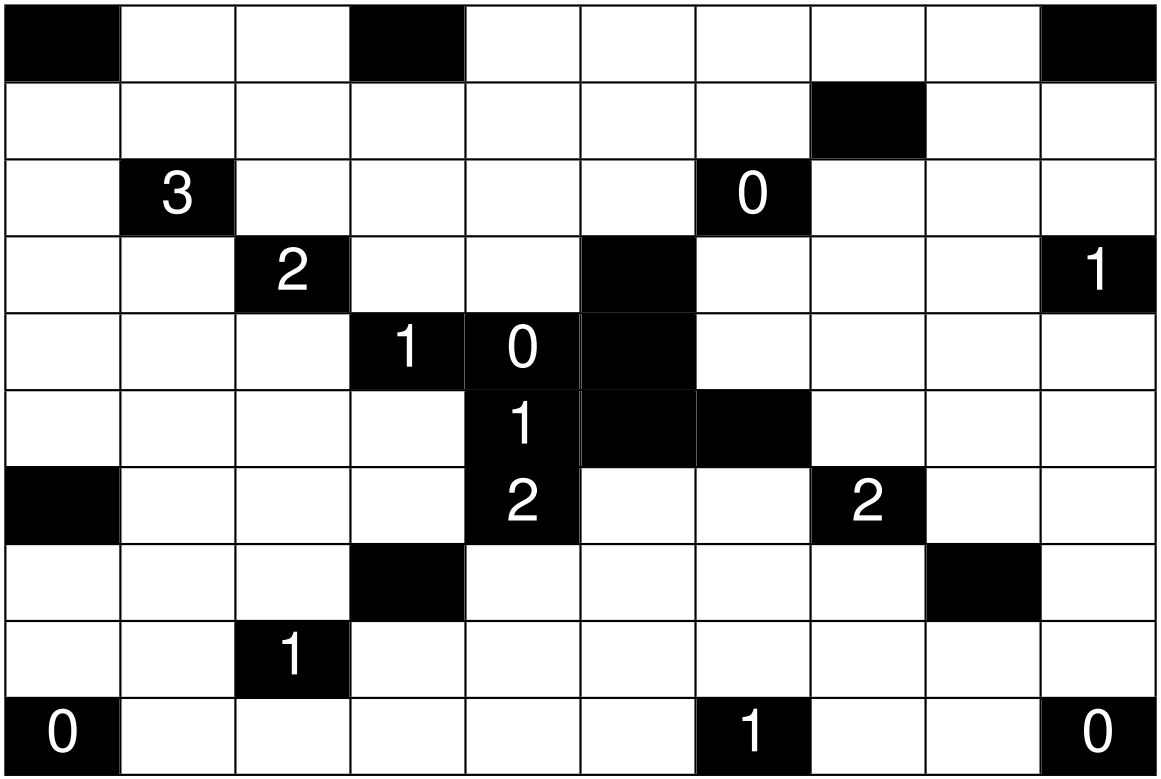
Problem



Rules

1. Lightup is played on a rectangular grid. Cells are either white or black, black cells may contain numbers from 0 to 4.
2. The aim of the game is to place lights in white cells so that they illuminate all white cells.
3. A light illuminates all cells in horizontal or vertical direction, until the light is blocked by a black cell.
4. Lights may not shine on each other.
5. A number in a black cell indicates how many lights are contained in neighboring white cells (sharing an edge, i.e. there are atmost 4 neighbors).

Solution

■	*		■	*					■
			*				■		
*	3	*				0			*
	*	2			■				1
			1	0	■	*			
			*	1	■	■	*		
■		*		2	*		2	*	
*			■	*				■	*
		1						*	
0		*				1	*		0

Outline

Finite Domain Model

	X_1	X_2		X_3	X_4	X_5	X_6	X_7	
X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}		X_{15}	X_{16}
X_{17}	3	X_{18}	X_{19}	X_{20}	X_{21}	0	X_{22}	X_{23}	X_{24}
X_{25}	X_{26}	2	X_{27}	X_{28}		X_{29}	X_{30}	X_{31}	1
X_{32}	X_{33}	X_{34}	1	0		X_{35}	X_{36}	X_{37}	X_{38}
X_{39}	X_{40}	X_{41}	X_{42}	1			X_{43}	X_{44}	X_{45}
	X_{46}	X_{47}	X_{48}	2	X_{49}	X_{50}	2	X_{51}	X_{52}
X_{53}	X_{54}	X_{55}		X_{56}	X_{57}	X_{58}	X_{59}		X_{60}
X_{61}	X_{62}	1	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}	X_{68}	X_{69}
0	X_{70}	X_{71}	X_{72}	X_{73}	X_{74}	1	X_{75}	X_{76}	0

Variables for white cells, all variables are 0/1

Finite Domain Model

	X_1	X_2		X_3	X_4	X_5	X_6	X_7	
X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}		X_{15}	X_{16}
X_{17}	3	X_{18}	X_{19}	X_{20}	X_{21}	0	X_{22}	X_{23}	X_{24}
X_{25}	X_{26}	2	X_{27}	X_{28}		X_{29}	X_{30}	X_{31}	1
X_{32}	X_{33}	X_{34}	1	0		X_{35}	X_{36}	X_{37}	X_{38}
X_{39}	X_{40}	X_{41}	X_{42}	1			X_{43}	X_{44}	X_{45}
	X_{46}	X_{47}	X_{48}	2	X_{49}	X_{50}	2	X_{51}	X_{52}
X_{53}	X_{54}	X_{55}		X_{56}	X_{57}	X_{58}	X_{59}		X_{60}
X_{61}	X_{62}	1	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}	X_{68}	X_{69}
0	X_{70}	X_{71}	X_{72}	X_{73}	X_{74}	1	X_{75}	X_{76}	0

Type1 $X_{43} + X_{51} + X_{59} + X_{50} = 2$

Finite Domain Model

	X_1	X_2		X_3	X_4	X_5	X_6	X_7	
X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}		X_{15}	X_{16}
X_{17}	3	X_{18}	X_{19}	X_{20}	X_{21}	0	X_{22}	X_{23}	X_{24}
X_{25}	X_{26}	2	X_{27}	X_{28}		X_{29}	X_{30}	X_{31}	1
X_{32}	X_{33}	X_{34}	1	0		X_{35}	X_{36}	X_{37}	X_{38}
X_{39}	X_{40}	X_{41}	X_{42}	1			X_{43}	X_{44}	X_{45}
	X_{46}	X_{47}	X_{48}	2	X_{49}	X_{50}	2	X_{51}	X_{52}
X_{53}	X_{54}	X_{55}		X_{56}	X_{57}	X_{58}	X_{59}		X_{60}
X_{61}	X_{62}	1	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}	X_{68}	X_{69}
0	X_{70}	X_{71}	X_{72}	X_{73}	X_{74}	1	X_{75}	X_{76}	0

Type2 $X_{26} + X_{33} + X_{40} + X_{46} + X_{54} + X_{62} + X_{70} + X_{53} + X_{55} \geq 1$

Finite Domain Model

	X_1	X_2		X_3	X_4	X_5	X_6	X_7	
X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}		X_{15}	X_{16}
X_{17}	3	X_{18}	X_{19}	X_{20}	X_{21}	0	X_{22}	X_{23}	X_{24}
X_{25}	X_{26}	2	X_{27}	X_{28}		X_{29}	X_{30}	X_{31}	1
X_{32}	X_{33}	X_{34}	1	0		X_{35}	X_{36}	X_{37}	X_{38}
X_{39}	X_{40}	X_{41}	X_{42}	1			X_{43}	X_{44}	X_{45}
	X_{46}	X_{47}	X_{48}	2	X_{49}	X_{50}	2	X_{51}	X_{52}
X_{53}	X_{54}	X_{55}		X_{56}	X_{57}	X_{58}	X_{59}		X_{60}
X_{61}	X_{62}	1	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}	X_{68}	X_{69}
0	X_{70}	X_{71}	X_{72}	X_{73}	X_{74}	1	X_{75}	X_{76}	0

Type3 $X_1 + X_2 \leq 1$

Finite Domain Model

	X_1	X_2		X_3	X_4	X_5	X_6	X_7	
X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}		X_{15}	X_{16}
X_{17}	3	X_{18}	X_{19}	X_{20}	X_{21}	0	X_{22}	X_{23}	X_{24}
X_{25}	X_{26}	2	X_{27}	X_{28}		X_{29}	X_{30}	X_{31}	1
X_{32}	X_{33}	X_{34}	1	0		X_{35}	X_{36}	X_{37}	X_{38}
X_{39}	X_{40}	X_{41}	X_{42}	1			X_{43}	X_{44}	X_{45}
	X_{46}	X_{47}	X_{48}	2	X_{49}	X_{50}	2	X_{51}	X_{52}
X_{53}	X_{54}	X_{55}		X_{56}	X_{57}	X_{58}	X_{59}		X_{60}
X_{61}	X_{62}	1	X_{63}	X_{64}	X_{65}	X_{66}	X_{67}	X_{68}	X_{69}
0	X_{70}	X_{71}	X_{72}	X_{73}	X_{74}	1	X_{75}	X_{76}	0

Type3 $X_3 + X_{12} + X_{20} + X_{28} \leq 1$

State after propagation

■	*		■	*					■
			*				■		
*	3	*				0			*
	*	2			■				1
			1	0	■	*			
			*	1	■	■	*		
■		*		2	*		2	*	
*			■	*				■	*
		1	.			.	.	.	
0		.	.			1	.	■	0

MIP/SAT Model

- Basically the same model as for finite domains
- MIP
  - Variables are 0/1
  - Objective function min $\sum X_i$ or max $\sum X_i$
  - Check LP relaxation for integrality
- SAT
  - Using Pseudo-Boolean extension to Minisat
  - No objective function
  - Automatic translation of (in)equalities into clauses

FD+search

Group	X	Y	K	Solved %		Stats			
				Setup	Total	Avg Time	Max Time	Avg Back	Max Back
mix-1	10	10	16	31.25	100.00	0.02	0.07	2.44	27
mix-1	17	17	8	12.50	100.00	0.05	0.17	14.75	100
mix-1	45	31	1	0.00	100.00	3.85	3.85	392.00	392
mix-2	10	10	16	37.50	100.00	0.01	0.02	0.62	3
mix-2	17	17	8	0.00	100.00	0.08	0.36	54.75	408
mix-2	45	31	1	0.00	100.00	1.25	1.25	201.00	201
nikoli-1	10	10	21	100.00	100.00	0.01	0.03	0.00	0
nikoli-1	18	10	25	36.00	100.00	0.02	0.05	0.84	7
nikoli-1	24	14	33	6.06	100.00	0.08	0.51	16.33	264
nikoli-1	36	20	17	5.88	94.12	28.91	408.06	10066.81	136552
nikoli-2	10	10	17	100.00	100.00	0.01	0.02	0.00	0
nikoli-2	18	10	27	48.15	100.00	0.02	0.04	1.00	13
nikoli-2	24	14	33	12.12	100.00	0.05	0.16	9.55	166
nikoli-2	36	20	19	5.26	100.00	36.95	594.68	10978.95	171636
All			242	33.06	99.59	4.88	594.68	1542.61	171636

Minisat+

Group	X	Y	K	Restart	Conflict	Avg Dec	Max Dec	Avg Time	Max Time
mix-1	10	10	16	1.00	2.50	14.19	49	0.01	0.02
mix-1	17	17	8	1.00	9.38	77.00	150	0.03	0.04
mix-1	45	31	1	1.00	73.00	926.00	926	0.16	0.16
mix-2	10	10	16	1.00	3.12	19.06	49	0.01	0.01
mix-2	17	17	8	1.00	10.62	83.75	160	0.03	0.04
mix-2	45	31	1	1.00	43.00	419.00	419	0.17	0.17
nikoli-1	10	10	21	1.00	0.00	1.00	1	0.01	0.01
nikoli-1	18	10	25	1.00	2.92	20.28	79	0.02	0.04
nikoli-1	24	14	33	1.00	8.79	73.67	223	0.03	0.05
nikoli-1	36	20	17	1.00	23.47	236.88	645	0.07	0.10
nikoli-2	10	10	17	1.00	0.00	1.00	1	0.01	0.02
nikoli-2	18	10	27	1.00	2.11	12.59	59	0.02	0.03
nikoli-2	24	14	33	1.00	7.85	69.67	173	0.04	0.06
nikoli-2	36	20	19	1.00	27.21	261.21	629	0.08	0.15
All			242	1.00	8.10	73.42	926	0.03	0.17

MILP (Coin-OR)

Group	X	Y	K	Vars	LP Relaxation		IP		
					Frac Min	Frac Max	Nodes	Avg Time	Max Time
mix-1	10	10	16	77.62	68.75	25.00	0.00	0.03	0.06
mix-1	17	17	8	218.25	12.50	12.50	0.00	0.08	0.11
mix-1	45	31	1	1023.00	0.00	0.00	0.00	0.95	0.95
mix-2	10	10	16	76.88	56.25	37.50	0.00	0.03	0.04
mix-2	17	17	8	221.62	12.50	0.00	0.00	0.09	0.11
mix-2	45	31	1	1070.00	0.00	0.00	0.00	0.79	0.79
nikoli-1	10	10	21	74.76	100.00	28.57	0.00	0.03	0.04
nikoli-1	18	10	25	135.76	56.00	8.00	0.00	0.04	0.07
nikoli-1	24	14	33	252.91	18.18	12.12	0.00	0.09	0.19
nikoli-1	36	20	17	546.35	5.88	5.88	0.00	0.25	0.66
nikoli-2	10	10	17	77.65	100.00	47.06	0.00	0.03	0.04
nikoli-2	18	10	27	136.37	74.07	7.41	0.00	0.04	0.06
nikoli-2	24	14	33	251.88	27.27	6.06	0.00	0.09	0.15
nikoli-2	36	20	19	550.68	26.32	0.00	0.00	0.32	0.85
All			242	225.04	47.52	14.88	0.00	0.10	0.95

Finite Set Model

```
1  intset(Set,1,70),
2  ...
3  #(Set /\ [43,50,51,59],2),
4  ...
5  X :: 1..inf,
6  #(Set /\ [26,33,40,46,53,54,55,62,70],X),
7  ...
8  Y1 :: 0..1,
9  #(Set /\ [1,2],Y1),
10 Y2 :: 0..1,
11 #(Set /\ [3,12,20,28],Y2),
12 ...
13 insetdomain(Set,any,small_first,in_notin),
```

Finite Set + Shaving

Group	X	Y	K	Solved %			Stats	
				Setup	Shave	Double	Avg Time	Max Time
mix-1	10	10	16	31.25	100.00	100.00	0.21	0.56
mix-1	17	17	8	12.50	75.00	100.00	2.34	4.56
mix-1	45	31	1	0.00	0.00	0.00	203.14	203.14
mix-2	10	10	16	37.50	93.75	100.00	0.28	0.66
mix-2	17	17	8	0.00	100.00	100.00	2.72	4.13
mix-2	45	31	1	0.00	0.00	100.00	174.55	174.55
nikoli-1	10	10	21	100.00	100.00	100.00	0.12	0.15
nikoli-1	18	10	25	36.00	100.00	100.00	0.54	1.13
nikoli-1	24	14	33	6.06	87.88	100.00	2.50	7.22
nikoli-1	36	20	17	5.88	88.24	100.00	19.43	43.70
nikoli-2	10	10	17	100.00	100.00	100.00	0.12	0.15
nikoli-2	18	10	27	48.15	100.00	100.00	0.45	0.99
nikoli-2	24	14	33	12.12	96.97	100.00	2.78	9.27
nikoli-2	36	20	19	5.26	73.68	100.00	19.24	64.15
All			242	33.06	92.98	99.59	5.48	203.14

What's going on?

- Finite Set model hopeless
- SAT reasoning not stronger than FD model
- Branching much better
  - Not just restart strategy
  - Variable selection/value selection more sophisticated
  - Clause (Nogood) learning
  - Primitives missing in CP systems

Outline

Shaving

- What-if analysis for values
- Test what happens if you assign a value
  - If it leads to inconsistency, remove value from domain
- Iterate over all variables, all values
- (Repeat until saturation)
- Controversy if this is a “fair” puzzle solving method

FD+shaving

Group	X	Y	K	Solved %			Stats	
				Setup	Shave	Double	Avg Time	Max Time
mix-1	10	10	16	31.25	93.75	100.00	0.02	0.12
mix-1	17	17	8	12.50	100.00	100.00	0.06	0.09
mix-1	45	31	1	0.00	100.00	100.00	0.62	0.62
mix-2	10	10	16	37.50	100.00	100.00	0.02	0.06
mix-2	17	17	8	0.00	87.50	100.00	0.08	0.14
mix-2	45	31	1	0.00	100.00	100.00	0.51	0.51
nikoli-1	10	10	21	100.00	100.00	100.00	0.01	0.03
nikoli-1	18	10	25	36.00	100.00	100.00	0.02	0.05
nikoli-1	24	14	33	6.06	96.97	100.00	0.05	0.16
nikoli-1	36	20	17	5.88	58.82	100.00	0.20	0.77
nikoli-2	10	10	17	100.00	100.00	100.00	0.01	0.02
nikoli-2	18	10	27	48.15	100.00	100.00	0.02	0.04
nikoli-2	24	14	33	12.12	90.91	100.00	0.05	0.11
nikoli-2	36	20	19	5.26	68.42	100.00	0.21	0.68
All			242	33.06	92.15	100.00	0.06	0.77

Introspection

- Humans solve puzzle like FD solver
- Propagate as far as possible
- Detect pattern in unsolved part
- Apply rule to remove/fix values
- Apply shaving when required

Examples of Rules

X		X
	3	
X		X

Table: Diagonal Constraint Pattern

Examples of Rules

		X	
		1	X
*	3		
	*		

Table: Combine Constraint Pattern

Examples of Rules

■	X	X			X	X	■
		■	1	2	*		
■	X	X			■		

Table: Neighbor Constraint Pattern

Examples of Rules

1		X	X		1
X					X
1		X	X		1

Table: Border Constraint Pattern

Examples of Rules

	X		X	
■	1	*	1	■
	■		■	

Table: Double One Constraint Pattern

Examples of Rules

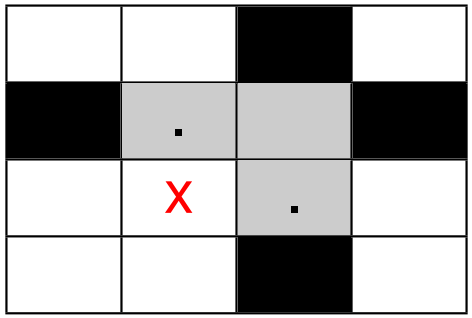


Table: L Constraint Pattern

Examples of Rules

	■	X		X		■
		*	2	■	2	*
■	X	X		X		■

Table: Bi Neighbor Constraint Pattern

Examples of Rules

	■	X		X	X	X		■
		■	2	*	■	■	2	*
■	X	X		X	X	X		■

Table: Long Neighbor Constraint Pattern

Examples of Rules

		X	X	X		
*	2		X		3	*
		X	X	X		

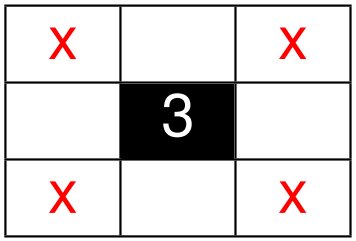
Table: Full Neighbor Constraint Pattern

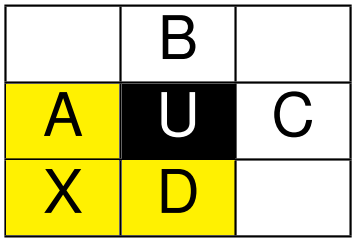
Examples of Rules

						■	
		X	X	X		1	■
*	2		X	X	X		
	■						

Table: Oblique Constraint Pattern

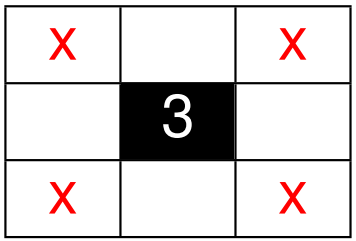
Explaining the Rule

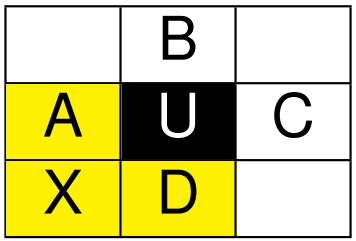




$$\begin{aligned} A + B + C + D &= U \\ A + X &\leq 1 \\ D + X &\leq 1 \end{aligned}$$

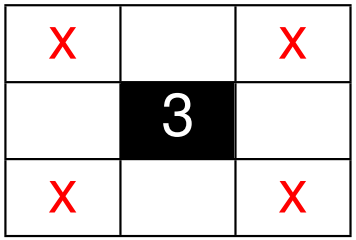
Explaining the Rule

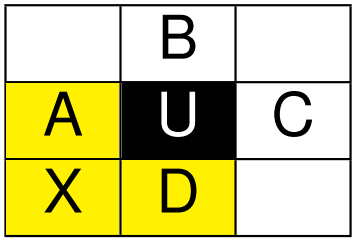




$$
\begin{aligned}
(*-1): \quad A+B+C+D &= U \\
(*1): \quad A+X &\leq 1 \\
(*1): \quad D+X &\leq 1 \\
\hline
-A-B-C-D+A+X+D+X &\leq -U+2
\end{aligned}
$$

Explaining the Rule





$$
\begin{aligned}
-A-B-C-D+A+X+D+X &\leq -U+2 \\
U+2*X &\leq 2+B+C \\
3+2*X &\leq 2+B+C \implies X=0
\end{aligned}
$$

FD+redundant constraints

Group	X	Y	K	Solved %				Stats	
				Setup	Redund	Shave	Double	Avg Time	Max Time
mix-1	10	10	16	31.25	100.00	100.00	100.00	0.03	0.19
mix-1	17	17	8	12.50	100.00	100.00	100.00	0.08	0.17
mix-1	45	31	1	0.00	100.00	100.00	100.00	1.33	1.33
mix-2	10	10	16	37.50	100.00	100.00	100.00	0.02	0.03
mix-2	17	17	8	0.00	75.00	100.00	100.00	0.11	0.20
mix-2	45	31	1	0.00	0.00	100.00	100.00	1.29	1.29
nikoli-1	10	10	21	100.00	100.00	100.00	100.00	0.02	0.07
nikoli-1	18	10	25	36.00	100.00	100.00	100.00	0.05	0.10
nikoli-1	24	14	33	6.06	96.97	100.00	100.00	0.12	0.29
nikoli-1	36	20	17	5.88	94.12	100.00	100.00	0.39	1.33
nikoli-2	10	10	17	100.00	100.00	100.00	100.00	0.02	0.03
nikoli-2	18	10	27	48.15	100.00	100.00	100.00	0.04	0.06
nikoli-2	24	14	33	12.12	96.97	100.00	100.00	0.12	0.53
nikoli-2	36	20	19	5.26	94.74	100.00	100.00	0.49	1.51
All			242	33.06	97.11	100.00	100.00	0.13	1.51

Open Questions

- Can we automate this efficiently?
- Does it find new rules which we didn't apply by hand?

Outline

Outline

Problem

	23	30			27	12	16
16				24 17			
17			29 15				
35						12	
	7			8 7			7
	11	16 10					
21					5		
6					3		

Rules

1. The puzzle uses a rectilinear grid of back and white cells. Black cells may contain hints (integer numbers). The number below the diagonal divider is the hint for cells below, the number above the diagonal divider is the hint for cells to the right.
2. The task is to enter numbers from 1 to 9 into the white cells satisfying the constraints.
3. The sum of a continuous block of white cells in horizontal (or vertical) direction must be equal to the hint given in the black cell to the left (above).
4. All numbers in a continuous block of white cells must be pairwise different.

Problem Solution

	23	30			27	12	16
16	9	7		24 17	8	7	9
17	8	9	29 15	8	9	5	7
35	6	8	5	9	7	12	
	7	6	1	8 7	2	6	7
	11	16 10	4	6	1	3	2
21	8	9	3	1	5	1	4
6	3	1	2		3	2	1

Outline

Finite Domain Model

Grid G: all white cells
Hints H: set of hints <I,v>
Variables x_i the value stored in cell i

$$\forall_{i \in G} : x_i \in [1, 9]$$

$$\forall_{<I,v> \in H} : \texttt{alldifferent}(\{x_i | i \in I\})$$

$$\forall_{<I,v> \in H} : \sum_{i \in I} x_i = v$$

MIP/SAT

0/1 variables $y_{i,j}$ indicate if cell i contains value j

$$\forall_{i \in G}, \forall_{j \in [1,9]} : y_{ij} \in \{0, 1\}$$

$$\forall_{i \in G} : \sum_{j \in [1,9]} y_{ij} = 1$$

$$\forall_{<I,v> \in H}, \forall_{j \in [1,9]} : \sum_{i \in I} y_{ij} \leq 1$$

$$\forall_{<I,v> \in H} : \sum_{i \in I} \sum_{j \in [1,9]} j * y_{ij} = v$$

FD + search

Set	X	Y	K	Solved %		Stats			
				Setup	Total	Avg Time	Max Time	Avg Back	Max Back
giant	32	42	3	0.00	0.00	n/a	n/a	n/a	n/a
giants	6	6	1	0.00	100.00	0.00	0.00	0.00	0
giants	32	22	14	0.00	42.86	6.09	13.55	18471.17	43509
mix	12	12	3	0.00	100.00	0.01	0.01	1.33	3
mix	16	16	18	0.00	100.00	0.16	0.91	425.56	3334
mix	32	22	2	0.00	100.00	7.22	14.28	16987.00	33622
mix2	12	12	3	0.00	100.00	0.01	0.01	3.00	5
mix2	16	16	18	0.00	100.00	0.34	2.30	1297.61	11732
mix2	32	22	2	0.00	50.00	5.12	5.12	9329.00	9329
puzzler	14	12	9	0.00	100.00	0.05	0.32	153.22	1307
suzuki	20	12	51	0.00	100.00	0.03	0.22	33.10	243
wh	12	10	2	0.00	100.00	0.01	0.01	0.00	0
wh	12	16	3	0.00	100.00	0.01	0.02	12.00	17
wh	20	14	1	0.00	100.00	0.02	0.02	10.00	10
wiki	8	8	1	0.00	100.00	0.01	0.01	3.00	3
All			131	0.00	90.84	0.56	14.28	1582.15	43509

FD + shaving

Set	X	Y	K	Solved %		Stats			
				Shave	Total	Avg Time	Max Time	Avg Back	Max Back
giant	32	42	3	0.00	66.67	14.53	15.82	33510.00	36702
giants	6	6	1	100.00	100.00	0.00	0.00	0.00	0
giants	32	22	14	28.57	57.14	1.63	6.08	3685.62	16933
mix	12	12	3	100.00	100.00	0.01	0.01	0.00	0
mix	16	16	18	88.89	100.00	0.09	0.63	89.78	1615
mix	32	22	2	50.00	100.00	0.89	1.51	908.50	1817
mix2	12	12	3	100.00	100.00	0.01	0.02	0.00	0
mix2	16	16	18	88.89	100.00	0.07	0.27	17.11	263
mix2	32	22	2	50.00	100.00	1.33	2.11	1782.50	3565
puzzler	14	12	9	88.89	100.00	0.03	0.05	0.11	1
suzuki	20	12	51	84.31	100.00	0.03	0.12	0.37	19
wh	12	10	2	100.00	100.00	0.01	0.01	0.00	0
wh	12	16	3	100.00	100.00	0.04	0.06	0.00	0
wh	20	14	1	100.00	100.00	0.03	0.03	0.00	0
wiki	8	8	1	100.00	100.00	0.01	0.01	0.00	0
All			131	78.63	94.66	0.42	15.82	837.35	36702

SAT

Set	X	Y	K	Solved %	Restart	Decisions Avg	Decisions Max	Time Avg	Time Max
giant	32	42	3	100.00	17.67	825987.67	920827	248.02	292.07
giants	6	6	1	100.00	3.00	932.00	932	0.09	0.09
giants	32	22	14	100.00	16.36	399448.50	817837	79.30	177.94
mix	12	12	3	100.00	7.33	11162.00	17509	1.03	1.33
mix	16	16	18	100.00	11.39	51620.39	101888	6.39	11.52
mix	32	22	2	100.00	15.50	266276.00	306874	43.93	47.42
mix2	12	12	3	100.00	8.00	12373.33	21390	1.36	1.90
mix2	16	16	18	100.00	10.72	43105.22	70487	5.32	7.37
mix2	32	22	2	100.00	15.50	278856.50	294193	38.42	45.09
puzzler	14	12	9	100.00	9.78	24022.11	37416	2.27	3.36
suzuki	20	12	51	100.00	9.76	30679.63	53245	3.58	6.66
wh	12	10	2	100.00	7.50	11139.50	14692	1.03	1.03
wh	12	16	3	100.00	9.67	25999.67	29044	2.02	2.13
wh	20	14	1	100.00	10.00	30437.00	30437	3.42	3.42
wiki	8	8	1	100.00	7.00	7244.00	7244	0.58	0.58
All			131	100.00	10.98	98136.83	920827	18.72	292.07

MIP (Cplex 10.1)

Group	X	Y	K	Nr Vars	Solved	Avg Time	Max Time
giant	32	42	3	9057.00	0.00	n/a	n/a
giants	6	6	1	162.00	100.00	0.01	0.01
giants	32	22	14	4527.64	14.29	4.31	5.98
mix	12	12	3	756.00	100.00	0.10	0.13
mix	16	16	18	1479.00	94.44	0.60	2.00
mix	32	22	2	4396.50	100.00	46.79	91.73
mix2	12	12	3	747.00	100.00	0.10	0.13
mix2	16	16	18	1499.50	100.00	4.24	66.86
mix2	32	22	2	4338.00	50.00	12.41	12.41
puzzler	14	12	9	938.00	100.00	0.56	3.84
wh	12	10	2	648.00	100.00	0.07	0.08
wh	12	16	3	1089.00	100.00	0.14	0.16
wh	20	14	1	1611.00	100.00	0.20	0.20
wiki	8	8	1	324.00	100.00	0.03	0.03
All			80	2265.64	78.75	3.30	91.73

Outline

Example after Initial Propagation

	23	30			27	12	16
16	 789	 789		24 17	... ..6 789	... ..6 789	 789
17	... ... .89	... ... .89	29 15	... ... .89	... .56 789	..3 456 ...	 789
35	... .56 78.	... ..6 789	123 456 789	... ... .89	123 456 789	12	
	7	..3 456 ...	123 4.. ...	8 7	123 456 7..	123 456 7..	7
	11	16 10	123 456 789	123 456 ...	123 456 789	123 456 789	123 45. ...
21	 789	 789	123 456 ...	123 456 ...	5	123 4.. ...	123 4.. ...
6	.23 4.. ...	123	123		3	12.	12. ... ...

Problem: Missing Propagation

```
[X1,X2,X3,X4,X5] :: 1..9,
alldifferent([X1,X2,X3,X4,X5]),
X1+X2+X3+X4+X5 #= 15
```

No domain reduction

$$\sum_{k \in I} x_k = N$$

$$\overline{x_i} = N - \sum_{k \in I, k \neq i} \underline{x_k}$$

$$\underline{x_i} = N - \sum_{k \in I, k \neq i} \overline{x_k}$$

Problem: Missing Propagation

```
[X1,X2,X3,X4,X5] :: 1..9,
alldifferent([X1,X2,X3,X4,X5]),
X1+X2+X3+X4+X5 #= 15
```

No domain reduction

$$\sum_{k \in I} x_k = 15$$

$$\overline{x_i} = 15 - \sum_{k \in I, k \neq i} 1 = 11$$

$$\underline{x_i} = 15 - \sum_{k \in I, k \neq i} 9 = -21$$

Problem: Missing Propagation

```
[X1,X2,X3,X4,X5] :: 1..9,
alldifferent([X1,X2,X3,X4,X5]),
X1+X2+X3+X4+X5 #= 15
```

Possible domain reduction

$$\sum_{k \in I} x_k = 15$$

$$\overline{x_i} = 15 - (1 + 2 + 3 + 4) = 5$$

$$\underline{x_i} = 15 - (5 + 4 + 3 + 2) = 1$$

Precompute Removable Values

Value	Arity							
	2	3	4	5	6	7	8	9
3	[3, 4, 5, 6, 7, 8, 9]	-	-	-	-	-	-	-
4	[2, 4, 5, 6, 7, 8, 9]	-	-	-	-	-	-	-
5	[5, 6, 7, 8, 9]	-	-	-	-	-	-	-
6	[3, 6, 7, 8, 9]	[4, 5, 6, 7, 8, 9]	-	-	-	-	-	-
7	[7, 8, 9]	[3, 5, 6, 7, 8, 9]	-	-	-	-	-	-
8	[4, 8, 9]	[6, 7, 8, 9]	-	-	-	-	-	-
9	[9]	[7, 8, 9]	-	-	-	-	-	-
10	[5]	[8, 9]	[5, 6, 7, 8, 9]	-	-	-	-	-
11	[1]	[9]	[4, 6, 7, 8, 9]	-	-	-	-	-
12	[1, 2, 6]	[]	[7, 8, 9]	-	-	-	-	-
13	[1, 2, 3]	[]	[8, 9]	-	-	-	-	-
14	[1, 2, 3, 4, 7]	[]	[9]	-	-	-	-	-
15	[1, 2, 3, 4, 5]	[]	[]	[6, 7, 8, 9]	-	-	-	-
16	[1, 2, 3, 4, 5, 6, 8]	[]	[]	[5, 7, 8, 9]	-	-	-	-
17	[1, 2, 3, 4, 5, 6, 7]	[]	[]	[8, 9]	-	-	-	-
18	-	[]	[]	[9]	-	-	-	-
19	-	[1]	[]	[]	-	-	-	-
20	-	[1, 2]	[]	[]	-	-	-	-
21	-	[1, 2, 3]	[]	[]	[7, 8, 9]	-	-	-
22	-	[1, 2, 3, 4]	[]	[]	[6, 8, 9]	-	-	-
23	-	[1, 2, 3, 4, 5, 7]	[]	[]	[9]	-	-	-
24	-	[1, 2, 3, 4, 5, 6]	[]	[]	[]	-	-	-
...	...	...	...	...	...	...	...	...

Example after Value Removal

	23	30			27	12	16
16	9	... ... 7.9		24 17	 789	 789	... ... 7.9
17	... ... .89	... ... .89	29 15	... ... .89	... .5. 789	... .5. 789	... ... 7.9
35	... ..6 .89	... ..6 789	5	... ... .89	... .56 789	12	
	7	6	123 45. ...	8 7	123 .56 7..	123 .56 ...	7
	11	16 10	123 4.. ...	123 4.6 ...	123 4.6 ...	123 4.6 ...	12. 4.. ...
21	.23 456 789	123 4.6 789	123 45. ...	123 456 ...	5	123 4.. ...	12. 4.. ...
6	.23	123	123		3	12.	12. ... ...

Example after Value Removal + Propagation

	23	30			27	12	16
16	9	7		24 17	8	7	9
17	8	9	29 15	8	9	5	7
35	6	8	5	9	7	12	
	7	6	1	8 7	2	6	7
	11	10	16 .23 4.. ...	... 4.6 ...	1	.23 4.. ...	.2. 4.. ...
21	... ... .89	... ... .89	.23 4.. ...	123	5	123	.2. 4.. ...
6	.23	12.	.23		3	12.	12. ... ...

FD + Removed values

Set	X	Y	K	Solved %			Time		Backtrack	
				Setup	Shave	Total	Avg	Max	Avg	Max
giant	32	42	3	0.00	100.00	100.00	0.18	0.20	0.00	0
giants	6	6	1	100.00	100.00	100.00	0.00	0.00	0.00	0
giants	32	22	14	0.00	100.00	100.00	0.09	0.14	0.00	0
mix	12	12	3	100.00	100.00	100.00	0.01	0.02	0.00	0
mix	16	16	18	27.78	100.00	100.00	0.02	0.07	0.00	0
mix	32	22	2	0.00	100.00	100.00	0.06	0.06	0.00	0
mix2	12	12	3	100.00	100.00	100.00	0.01	0.02	0.00	0
mix2	16	16	18	16.67	100.00	100.00	0.02	0.04	0.00	0
mix2	32	22	2	0.00	100.00	100.00	0.05	0.05	0.00	0
puzzler	14	12	9	44.44	100.00	100.00	0.01	0.02	0.00	0
suzuki	20	12	51	1.96	86.27	100.00	0.02	0.06	0.00	0
wh	12	10	2	0.00	100.00	100.00	0.01	0.01	0.00	0
wh	12	16	3	0.00	100.00	100.00	0.01	0.02	0.00	0
wh	20	14	1	0.00	100.00	100.00	0.02	0.02	0.00	0
wiki	8	8	1	0.00	100.00	100.00	0.00	0.00	0.00	0
All			131	15.27	94.66	100.00	0.03	0.20	0.00	0

Arc-consistent alldifferent-sum

- There is still missed propagation
- We need arc-consistent alldifferent-sum
- Every value for variable in constraint is supported
  - No such constraint in global constraint catalog
  - Write new global constraint? No...
  - Simulate by other global constraint
  - Use general mechanism

GCC with Cost

- Model proposed by M. Carlsson
- Generalized cardinality constraint
- How often values can occur in set of variables
- Cost matrix defines cost of assigning value to variable
- Can simulate alldifferent-sum
- Simulation not arc consistent

Propia

- Generalized Propagation (Le Provost & Wallace, 92)
- Deduce domain restrictions from all possible solutions
- Arc consistent for explicit representation
- In that case equivalent to *table* constraint

Counting Valid Tuples

Value	Arity 2	3	4	5	6	7	8	9
3	2	-	-	-	-	-	-	-
4	2	-	-	-	-	-	-	-
5	4	-	-	-	-	-	-	-
6	4	6	-	-	-	-	-	-
7	6	6	-	-	-	-	-	-
8	6	12	-	-	-	-	-	-
9	8	18	-	-	-	-	-	-
10	8	24	24	-	-	-	-	-
11	8	30	24	-	-	-	-	-
12	6	42	48	-	-	-	-	-
13	6	42	72	-	-	-	-	-
14	4	48	120	-	-	-	-	-
15	4	48	144	120	-	-	-	-
16	2	48	192	120	-	-	-	-
17	2	42	216	240	-	-	-	-
18	-	42	264	360	-	-	-	-
19	-	30	264	600	-	-	-	-
20	-	24	288	720	-	-	-	-
21	-	18	264	960	720	-	-	-
22	-	12	264	1080	720	-	-	-
23	-	6	216	1320	1440	-	-	-
24	-	6	192	1320	2160	-	-	-

Counting Valid Tuples (2)

Value	Arity							
	2	3	4	5	6	7	8	9
25	-	-	144	1440	2880	-	-	-
26	-	-	120	1320	3600	-	-	-
27	-	-	72	1320	5040	-	-	-
28	-	-	48	1080	5040	5040	-	-
29	-	-	24	960	5760	5040	-	-
30	-	-	24	720	5760	10080	-	-
31	-	-	-	600	5760	10080	-	-
32	-	-	-	360	5040	15120	-	-
33	-	-	-	240	5040	15120	-	-
34	-	-	-	120	3600	20160	-	-
35	-	-	-	120	2880	20160	-	-
36	-	-	-	-	2160	20160	40320	-
37	-	-	-	-	1440	15120	40320	-
38	-	-	-	-	720	15120	40320	-
39	-	-	-	-	720	10080	40320	-
40	-	-	-	-	-	10080	40320	-
41	-	-	-	-	-	5040	40320	-
42	-	-	-	-	-	5040	40320	-
43	-	-	-	-	-	-	40320	-
44	-	-	-	-	-	-	40320	-
45	-	-	-	-	-	-	-	362880

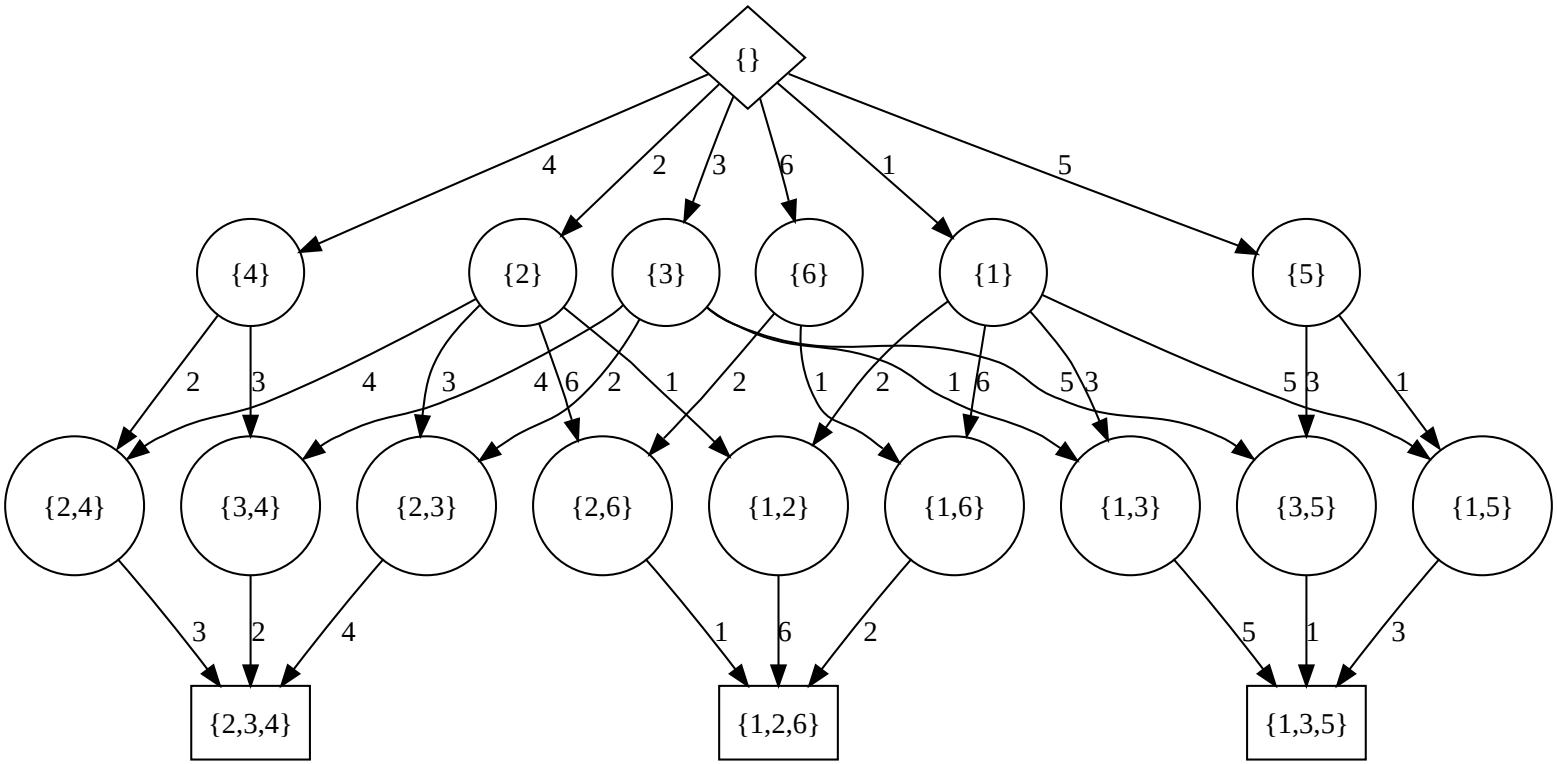
FD+ Propia

Set	X	Y	K	Solved %			Time		Back	
				Setup	Shave	Total	Avg	Max	Avg	Max
giant	32	42	3	100.00	100.00	100.00	1.12	1.67	0.00	0
giants	6	6	1	100.00	100.00	100.00	0.00	0.00	0.00	0
giants	32	22	14	100.00	100.00	100.00	1.78	8.95	0.00	0
mix	12	12	3	100.00	100.00	100.00	0.02	0.02	0.00	0
mix	16	16	18	100.00	100.00	100.00	0.10	0.16	0.00	0
mix	32	22	2	100.00	100.00	100.00	0.55	0.87	0.00	0
mix2	12	12	3	100.00	100.00	100.00	0.03	0.03	0.00	0
mix2	16	16	18	100.00	100.00	100.00	0.37	4.56	0.00	0
mix2	32	22	2	100.00	100.00	100.00	0.42	0.44	0.00	0
puzzler	14	12	9	100.00	100.00	100.00	0.05	0.11	0.00	0
suzuki	20	12	51	84.31	86.27	100.00	0.07	0.21	0.00	0
wh	12	10	2	100.00	100.00	100.00	0.05	0.07	0.00	0
wh	12	16	3	100.00	100.00	100.00	0.05	0.06	0.00	0
wh	20	14	1	100.00	100.00	100.00	0.09	0.09	0.00	0
wiki	8	8	1	100.00	100.00	100.00	0.01	0.01	0.00	0
All			131	93.89	94.66	100.00	0.33	8.95	0.00	0

More Compact Representation

- Arity 8 and 9 can be handled by alldifferent + domain reduction
- Tuple Compression (Katsirelos & Walsh, 2007)
  - Does not work for permutations
- Case Constraint (Beldiceanu & Carlsson)
- Regular Constraint (Pesant, 2004)
  - Accepts tuples which fit regular expression
  - Arc-consistent propagation
  - Just state disjunction of valid tuples
  - Generate compact automaton
  - Gecode model provided by C. Schulte

Automaton (3 variables with sum 9)



Is this enough?

- For many instances, arc-consistent alldifferent-sum + propagation is enough
- Found one collection which requires more reasoning (Suzuki)
- Similar experience with Sudoku
  - Complex interaction to be discovered

(Current) Best ECLiPSe Variant

FD + Reordering + Propia + Removal + Redundant Model + Shaving

Set	X	Y	K	Solved %			Time		Backtrack	
				Setup	Shave	Total	Avg	Max	Avg	Max
giant	32	42	3	100.00	100.00	100.00	0.30	0.32	0.00	0
giants	6	6	1	100.00	100.00	100.00	0.00	0.00	0.00	0
giants	32	22	14	100.00	100.00	100.00	0.34	1.34	0.00	0
mix	12	12	3	100.00	100.00	100.00	0.01	0.02	0.00	0
mix	16	16	18	100.00	100.00	100.00	0.04	0.07	0.00	0
mix	32	22	2	100.00	100.00	100.00	0.09	0.11	0.00	0
mix2	12	12	3	100.00	100.00	100.00	0.01	0.02	0.00	0
mix2	16	16	18	100.00	100.00	100.00	0.04	0.07	0.00	0
mix2	32	22	2	100.00	100.00	100.00	0.09	0.10	0.00	0
puzzler	14	12	9	100.00	100.00	100.00	0.03	0.04	0.00	0
suzuki	20	12	51	84.31	86.27	100.00	0.04	0.10	0.00	0
wh	12	10	2	100.00	100.00	100.00	0.01	0.01	0.00	0
wh	12	16	3	100.00	100.00	100.00	0.03	0.03	0.00	0
wh	20	14	1	100.00	100.00	100.00	0.03	0.03	0.00	0
wiki	8	8	1	100.00	100.00	100.00	0.00	0.00	0.00	0
All			131	93.89	94.66	100.00	0.07	1.34	0.00	0

Outline

Generating Puzzles

- For some puzzle types, this is easy (Sudoku, Lightup)
- Generating Kakuro puzzles is more difficult
- No information published
- Significant commercial interest

Idea 1: Generating alldifferent

- Take grid from existing puzzle or create your own
- State alldifferent constraints for each block
- Find solution
- Add numbers in block and use as hint
- Creates many *valid* puzzles
- They are very rarely *well posed*

Idea 2: Local search + CP

- Begin with alldifferent puzzle from idea 1
- Do local search minimizing the uncertainty in puzzle
- Count total domain size after propagation
- Neighborhood: Change one horizontal and one vertical constraint together by same value
- Use domains after propagation to select good candidates
- Converges in relatively few steps
- There are many well posed puzzles for a given grid

Local Search Example

	20	20			17	12	12
12	..3 45. 789	..3 45. 789		17 11	2	... ... 78.	... ... 78.
12	..3 45. 789	..3 45. 789	12 25	2	1	... 45. ...	... 45. ...
29	..3 456 78.	123 456 78.	123 456 78.	9	..3 45. ...	23	
	8	123 .56 7..	123 .56 7..	15 8	... ..6 7..	... ... .89	17
	12	29 12	..3 456 789	123 .56 7..	..3 45. ...	.23 456 7..	... .56 789
20	..3 45. 789	..3 45. 789	123 456 789	123 .56 7..	5	123 4.. ...	123 4.. ...
15	..3 45. 789	..3 45. 789	123 456 78.		15	... ..6 789	... ..6 789

Table: wiki1 Step 1 Cost 131

Local Search Example

	20	20			17	12	12
12	. . 3 45 . 789	. . 3 45 . 789		17 11	2	. . . . . . 78 .	. . . . . . 78 .
12	. . 3 45 . 789	. . 3 45 . 789	12 25	2	1	. . . 45	. . . 45 . . . .
29	. . 3 456 78 .	123 456 78 .	123 456 78 .	9	. . . 456 . . .	12	
	8	123 . 56 7 . .	123 . 56 7 . .	4 8	3	1	17
	12	29 12	 89	. . . . 56 7 . .	. . . . 56 7 . .	. 23	. . . . 56 . . .
20	. . 3 45 . 789	. . 3 45 . 789	123 456 789	123	5	. 23	. 23 . . . . . .
15	. . 3 45 . 789	. . 3 45 . 789	123 456 78 .		15	6	9

Table: wiki1 Step 2 Cost 101

Local Search Example

	20	20			17	12	12
12	..3 45. 789	..3 45. 789		17 11	2	... ... 78.	... ... 78.
12	..3 45. 789	..3 45. 789	12 25	2	1	... 45. ...	... 45. ...
29	..3 456 78.	123 456 78.	123 456 78.	9	... 4.6 ...	24	
	8	123 .56 7..	123 .56 7..	4 8	3	1	17
	12	29 12	9	... .5. 7..	... .5. 7..	6	2
20	..3 45. 789	..3 45. 789	123 456 78.	1.3	17	... ... .89	... ... .89
15	..3 45. 789	..3 45. 789	123 456 78.		15	... ... .89	... ..6 7..

Table: wiki1 Step 3 Cost 95

Local Search Example

	20	10			17	12	12
12	89	..3 4.. . . .		17 11	2	. . . . . . 78.	. . . . . . 78.
12	89	..3 4.. . . .	12 25	2	1	. . . 45. . . .	. . . 45. . . .
19	3	12.	12.	9	4	24	
	8	12.	. . . ..6 7..	4 8	3	1	17
	12	29 12	9	5	7	6	2
20	. . . 4.. 789	. . . 4.. 789	12. .56 . . .	3	17	89	89
15	..3 45. .8.	..3 45. .8.	.23 ..6 7..		15	89	. . . ..6 7..

Table: wiki1 Step 4 Cost 34

Local Search Example

	20	10			17	12	12
12	89	..3 4.. . . .		17 11	2	. . . . . . 78.	. . . . . . 78.
12	89	..3 4.. . . .	12 34	2	1	. . . 45. . . .	. . . 45. . . .
28	3	2	8	9	6	24	
	8	1	7	4 8	3	1	17
	12	29 12	9	7	5	6	2
20	. . . 4.. 789	. . . 4.. 789	. . . 4.6 . . .	1	17	89	89
15	..3 45. .8.	..3 45. .8.	. . . 4.6 . . .		15	89	. . . ..6 7..

Table: wiki1 Step 5 Cost 26

Local Search Example

	20	10			17	12	12
12	89	.3 4.. . . .		17 11	2	. . . . . . 78.	. . . . . . 78.
12	89	.3 4.. . . .	12 34	2	1	. . . 45. . . .	. . . 45. . . .
28	3	2	8	9	6	12	
	8	1	7	4 8	3	1	17
	12	29 12	9	7	5	2	6
20	. . . 4.. 789	. . . 4.. 789	. . . 4.6 . . .	1	5	3	2
15	.3 45. .8.	.3 45. .8.	. . . 4.6 . . .		15	6	9

Table: wiki1 Step 6 Cost 22

Local Search Example

	11	10			17	12	12
12	8	4		17 11	2	. . . . . 78.	. . . . . 78.
3	1	2	12 34	2	1	. . . 45. . .	. . . 45. . .
28	2	3	8	9	6	12	
	8	1	7	4 8	3	1	17
	12	29 12	9	7	5	2	6
20	. . . 4. . 789	. . . 4. . 789	. . . 4.6 . . .	1	5	3	2
15	. .3 45. .8.	. .3 45. .8.	. . . 4.6 . . .		15	6	9

Table: wiki1 Step 7 Cost 18

Local Search Example

	11	10			17	12	12
12	8	4		17 11	2	. . . . . 78.	. . . . . 78.
3	1	2	12 34	2	1	. . . 45. . .	. . . 45. . .
28	2	3	8	9	6	12	
	8	1	7	4 8	3	1	17
	12	29 5	9	7	5	2	6
20	9	4	6	1	5	3	2
8	3	1	4		15	6	9

Table: wiki1 Step 8 Cost 4

Local Search Example

	11	10			17	12	5
12	8	4		10 11	2	7	1
3	1	2	12 34	2	1	5	4
28	2	3	8	9	6	12	
	8	1	7	4 8	3	1	17
	12	29 5	9	7	5	2	6
20	9	4	6	1	5	3	2
8	3	1	4		15	6	9

Table: wiki1 Step 9 Cost 0

Outline

Conclusions

- Naive SAT more stable than naive CP on examples shown
  - No single explanation for this
  - Better (packaged) search needed
  - Unexploited constraint interaction
- CP = The Science of alldifferent?
  - Can we tell a priori when CP will work (better than SAT/MIP)?
  - How to choose best model without implementing all alternatives?