Uncertainty in Supply Chain Networks

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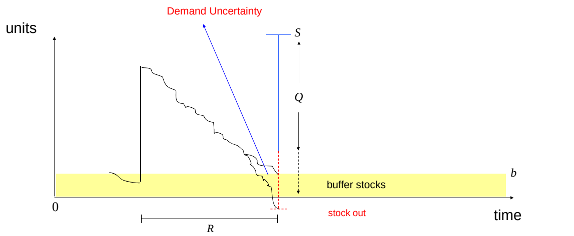
Determining the optimal inventory control policy parameters is key to profitability for any company involved in distribution and/or production of goods

We present a CP model to find the optimal dynamic (R,S) inventory policy parameters which minimize the expected cost when demand is stochastic and non-stationary, a minimum service level is required and a discrete stochastic lead-time is given

The (R,S) policy

- ❑ R → set of periods when orders have to be placed
- ❑ S → set of order-up-to-level values for the periods in R



Non-stationary Stochastic Lot Sizing

- ❑ The target is to find the **optimal (R,S) inventory policy** minimizing the total expected cost and meeting the required service level (stockouts %)
- ❑ Decisions have to be taken about the **periods when orders are fixed** and about the **size of such orders**
- ❑ Silver (1978): obtaining the optimal solution for the stochastic non-stationary formulation of the lot-sizing problem **requires significant computational efforts**.

MIP - approach

min $E\{TC\} = \sum_{t=1}^{N}(a\delta_t + h\tilde{I}_t)$

st. $(t = 1,\dots,N)$

- ❑ no buy-back constraint: $\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} \geq 0$
- ❑ replenishment condition: $\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} \leq M\delta_t$
- ❑ service level constraints:

$$\tilde{I}_t \geq \sum_{j=1}^{t}\left(G^{-1}_{d_{t-j+1}+d_{t-j+2}+\dots+d_t}(\alpha) - \sum_{k=t-j+1}^{t}\tilde{d}_k\right)P_{tj}$$

$$\sum_{j=1}^{t} P_{tj} = 1 \qquad j = 1,\dots,t$$

$$P_{tj} \geq \delta_{t-j+1} - \sum_{k=t-j+2}^{t}\delta_k \qquad j = 1,\dots,t$$

$\delta_t, P_{tj} \in \{0,1\}$

$\tilde{I}_t \geq 0$

CP - approach

- ❑ Stochastic programming model

Objective Function:

- ❑ Minimize total expected cost → E[TC] = holding cost + ordering cost

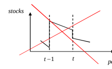
min $E[TC] = \sum_{t=1}^{N}\left(a\delta_t + h\tilde{I}_t\right)$

Decision Variables:

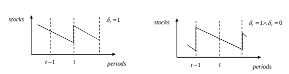
- ❑ Place an order at period *t* → $\delta_t \in \{0,1\}$
- ❑ Stocks level at period *t* → $\tilde{I}_t \in Z^+ \cup \{0\}$

Constraints: $t = 1,\dots,N$

- ❑ no buy-back constraint
  $\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} \geq 0$



- ❑ replenishment condition
  $\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} > 0 \Rightarrow \delta_t = 1$



- ❑ service level constraint
  $\tilde{I}_t \geq \Phi[t, \max_{j\in[1..t]} j\delta_j]$

It enforces a **buffer stock** at the end of each replenishment cycle such that **the required service level is met**

- ❑ extension for stochastic lead-time - **global chance-constraint**

$bufferConstr(\tilde{I}_1,\dots,\tilde{I}_N,\delta_1,\dots,\delta_N,d_1,\dots,d_N,l_1,\dots,l_T,\alpha)$

Selected Publications:

R. Rossi, S. A. Tarim, B. Hnich and S. Prestwich, "A Global Chance-Constraint for Stochastic Inventory Systems under Service Level Constraints", **Constraints: An International Journal**, to appear (2008)

S. A. Tarim, B. Hnich, R. Rossi and S. Prestwich, "Cost-based Filtering Techniques for Stochastic Inventory Control under Service Level Constraints", **Constraints: An International Journal**, to appear (2008)

S. A. Tarim, B. Hnich, R. Rossi and S. Prestwich, "Cost-Based Filtering for Stochastic Inventory Control", *Recent Advances in Constraints: 11th Annual ERCIM International Workshop on Constraint Solving and Constraint Logic Programming, CSCLP 2006 Caparica, Portugal, June 26-28, 2006 Revised Selected and Invited Papers*, **Lecture Notes in Computer Science**, Springer-Verlag, LNCS 4651, pp.169-183, 2007

R. Rossi, S. A. Tarim, B. Hnich and S. Prestwich, "Replenishment Planning for Stochastic Inventory Systems with Shortage Cost", *In proceedings of The Fourth International Conference on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems (CP-AI-OR 07) May 23-26, 2007 - Brussels, Belgium*, **Lecture Notes in Computer Science**, Springer-Verlag, LNCS 4510, pp.229-243, 2007

Tarim, S. A. and B. M. Smith , "Constraint Programming for Computing Non-Stationary (R,S) Policy", **European Journal of Operational Research**, forthcoming.

S. A. Tarim, S. Manandhar and T. Walsh, "Stochastic Constraint Programming: A Scenario-Based Approach", **Constraints: An International Journal**, Vol.11, pp.53-80, 2006.

Tarim, S. A. and B. G. Kingsman , "Modelling and Computing (Rn,Sn) Policies for Inventory Systems with Non-Stationary Stochastic Demand", **European Journal of Operational Research**, 174, pp.581-599, 2006.

CP – MIP comparison

CP model	*MIP model*
❑ $2N$ constraints	❑ $(N^2+5N)/2$ constraints
❑ $3N$ decision variables	❑ $(N^2+9N)/2$ decision variables
❑ Explores more nodes in the search tree	❑ Explores less nodes in the search tree
❑ Non-linear → uses the Element global constraint	❑ Linear

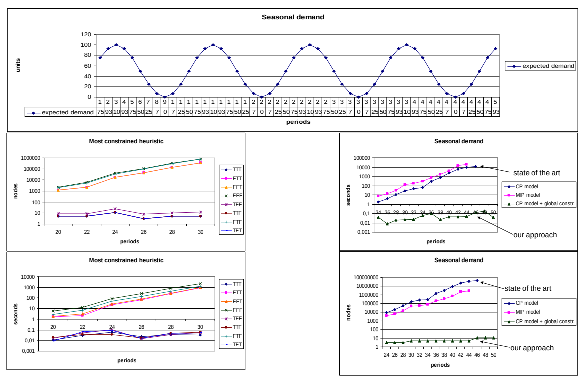
Filtering methods

Inventory levels I_t can assume only a finite set of feasible values. Thus we can reduce the domains whenever a partial solution is given.

We developed three global constraints which can be enforced at each node of the search tree:

- ❑ **Dynamic programming relaxation**: relaxes all the three constraint and build an Shortest Path Problem instance where every arc represents a replenishment cycle and its cost is equal to the respective replenishment cycle cost (ordering cost + holding cost);
- ❑ **Merging lemma**: builds a smaller instance merging two periods when no replenishment ($\delta_t = 0$) has been fixed in the second one. It is possible to infer more inconsistent values using the smaller smaller instance to get a further domain reduction for the original one;
- ❑ **Upper bounds tightening**: exploits the properties of a given partial solution to tighten the UB for the replenishment cycles length or to rule out those values related to replenishment cycles that are not in such solution.

Empirical results



Implemented using Java (JDK 1.5) and Choco (open source java solver)

Screenshots

- ❑ The software developed has been selected as one of the four candidates at ISA Award for **The Most Commercially Viable Software 2006**

- ❑ By means of **dedicated domain reduction techniques** and **cost-based filtering methods** we can now solve **real world inventory problems** over a non-trivial planning horizon
- ❑ We are able to **optimize under multiple level of uncertainty**: stochastic demand and lead time
- ❑ Lucent Tech. is currently **testing our software in its production planning business unit**

Acknowledgements

This research is supported by Science Foundation Ireland under Grant No. 03/CE3/I405 as part of the Centre for Telecommunications Value-Chain-Driven Research (CTVR) and under Grant No. 00/PI.1/C075